MA26600 Exam 2

GREEN VERSION

NAME: ____________________

PUID (10 digits): ____________________

INSTRUCTOR: ____________________

SECTION/TIME: ____________________

1. Fill in your NAME, your PUID (10 digits), your INSTRUCTOR's name and SECTION number (or class meeting TIME) above. Please print legibly and use your name as it appears in the Purdue directory; please don't shorten or use a nickname.
2. Make sure you have all 9 pages of the exam book.
3. There are 10 questions, each worth 10 points.
4. Questions 1–7 are multiple-choice questions. Indicate your choice of an answer by **circling the letter** next to the choice like this:

   D. My choice of a correct answer.

   Show your work on the question sheets in the space provided after each problem. Although no partial credit will be given on the multiple choice questions, any disputes about grades or grading will be settled by examining your written work on the question sheets.
5. Questions 8–10 are handwritten problems. **Write the solutions of the handwritten problems clearly and explain all steps.** You can use the back of the test pages for the scratch paper but it will not be looked for grading.
6. **NO CALCULATORS, BOOKS, NOTES, PHONES, OR CAMERAS ARE ALLOWED**.

   Turn off or put away all electronic devices.

1. If the method of undetermined coefficients is to be used on

$$y'' - 2y' + 5y = x\sin(2x),$$

which one of the following is the correct form for a particular solution y_p?

A. $y_p(x) = Ax^2\sin(2x) + Bx^2\cos(2x)$

B. $y_p(x) = Ax\sin(2x)$

C. $y_p(x) = Ax\sin(2x) + Bx\cos(2x)$

D. $y_p(x) = (Ax + B)\sin(2x) + (Cx + D)\cos(2x)$

E. $y_p(x) = (Ax^2 + Bx)\sin(2x) + (Cx^2 + Dx)\cos(2x)$

2. Transform the given differential equation

$$x'' + 2x' - 2x = \cos(4t)$$

into an equivalent system of first-order differential equations.

A. $x_1' = 2x_2, \quad x_2' = x_1 + x_2 + \cos(4t)$

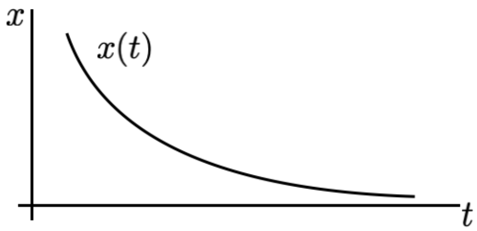
B. $x_1' = 2x_2, \quad x_2' = x_1 - x_2 + \cos(4t)$

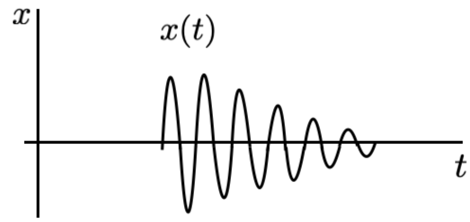
C. $x_1' = x_2, \quad x_2' = \cos(4t)$

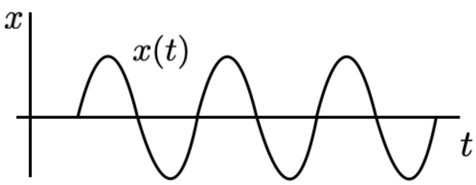
D. $x_1' = x_2, \quad x_2' = -2x_1 + 2x_2 - \cos(4t)$

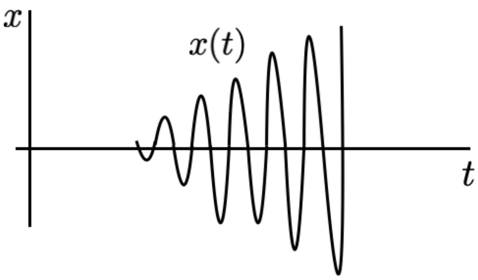
E. $x_1' = x_2, \quad x_2' = 2x_1 - 2x_2 + \cos(4t)$

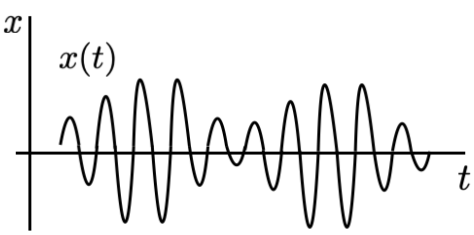
3. A damped mass-spring system is set in motion and undergoes an external force of the form $F(t) = \cos(\omega t)$. Which of the following plots represents the qualitative behavior of the displacement $x(t)$ of the mass **after a long time has passed**?

A. 

B. 

C. 

D. 

E. 

4. Consider the linear system of differential equations

$$\mathbf{x}' = \begin{bmatrix} 1 & -\alpha \\ 4 & -1 \end{bmatrix} \mathbf{x}.$$

For what values of the parameter α the origin is a saddle point for this system?

A. $\frac{1}{4} < \alpha < 4$

B. $\alpha < \frac{1}{4}$

C. $-4 < \alpha < 0$

D. $\alpha > 4$

E. $\alpha < 0$

5. Let $\mathbf{x}(t) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ be the particular solution to the initial value problem

$$\mathbf{x}'(t) = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \mathbf{x}(t), \quad \mathbf{x}(0) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}.$$

Find $x_1(1)$.

A. $-\dfrac{3}{2}e^{-1} + \dfrac{1}{2}e^{3}$

B. $-e\cos(2) + 2e\sin(2)$

C. $\dfrac{1}{2}e^{-1} - \dfrac{3}{2}e^{3}$

D. $2e\cos(2) - e\sin(2)$

E. $\dfrac{5}{2}e^{-2}$

6. Solve the initial value problem

$$\mathbf{x}' = \begin{bmatrix} -1 & -1 \\ 9 & -1 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 9 \end{bmatrix}.$$

A. $\mathbf{x} = e^{-t} \begin{bmatrix} \cos 3t - 3\sin 3t \\ 9\cos 3t - 3\sin 3t \end{bmatrix}$

B. $\mathbf{x} = e^{-t} \begin{bmatrix} \cos 3t - 3\sin 3t \\ 9\cos 3t + 3\sin 3t \end{bmatrix}$

C. $\mathbf{x} = e^{-t} \begin{bmatrix} \cos 3t + 3\sin 3t \\ 9\cos 3t - 3\sin 3t \end{bmatrix}$

D. $\mathbf{x} = e^{-t} \begin{bmatrix} \cos 3t + 3\sin 3t \\ 9\cos 3t + 3\sin 3t \end{bmatrix}$

E. $\mathbf{x} = e^{-t} \begin{bmatrix} \cos 3t + 3\sin 3t \\ -9\cos 3t + 3\sin 3t \end{bmatrix}$

7. Given that $\mathbf{x}_1(t) = \begin{bmatrix} e^{4t} \\ -e^{4t} \end{bmatrix}$ is a solution of the system of differential equations,

$$\mathbf{x}' = \begin{bmatrix} 3 & -1 \\ 1 & 5 \end{bmatrix} \mathbf{x},$$

find another solution $\mathbf{x}_2(t)$ of the system such that $\mathbf{x}_1(t)$ and $\mathbf{x}_2(t)$ are linearly independent.

A. $\mathbf{x}_2(t) = \begin{bmatrix} (t+1)e^{4t} \\ -te^{4t} \end{bmatrix}$

B. $\mathbf{x}_2(t) = \begin{bmatrix} 2e^{-4t} \\ e^{-4t} \end{bmatrix}$

C. $\mathbf{x}_2(t) = \begin{bmatrix} \cos(4t) \\ \sin(4t) \end{bmatrix}$

D. $\mathbf{x}_2(t) = \begin{bmatrix} te^{4t} \\ -te^{4t} \end{bmatrix}$

E. $\mathbf{x}_2(t) = \begin{bmatrix} (t-2)e^{4t} \\ (1-t)e^{4t} \end{bmatrix}$

8. The homogeneous differential equation

$$y'' - \frac{3}{x}y' + \frac{4}{x^2}y = 0, \quad x > 0$$

has two solutions $y_1(x) = x^2$ and $y_2(x) = x^2 \ln(x)$. Use the method of variation of parameters to find a particular solution of the nonhomogeneous differential equation

$$y'' - \frac{3}{x}y' + \frac{4}{x^2}y = 2, \quad x > 0.$$

9. Consider the linear system $\mathbf{x}' = \mathbf{A}\mathbf{x}$, where $\mathbf{A} = \begin{bmatrix} -5 & 1 \\ 4 & -2 \end{bmatrix}$.

(a) Find two linearly independent solutions of the system.

(b) Use the solutions found in part (a) to calculate the exponential matrix $e^{\mathbf{A}t}$.

10. Consider the first-order system

$$x' = y, \quad y' = 3x \quad \text{with} \quad x(0) = 4,\ y(0) = 2\sqrt{3}.$$

Find the solution and show that its trajectory is a branch of a hyperbola.
(Hint: Calculate $x^2 - y^2/3$ along the solution.)