MA26600 Exam 2

GREEN VERSION

NAME: ______________________________

PUID (10 digits): ______________________________

INSTRUCTOR: ______________________________

SECTION/TIME: ______________________________

1. Fill in your NAME, your PUID (10 digits), your INSTRUCTOR's name and SECTION number (or class meeting TIME) above. Please print legibly and use your name as it appears in the Purdue directory; please don't shorten or use a nickname.
2. Make sure you have all 8 pages of the exam book.
3. There are 10 questions, each worth 10 points.
4. Questions 1–7 are multiple-choice questions. Indicate your choice of an answer by **circling the letter** next to the choice like this:

   D. My choice of a correct answer.

   Show your work on the question sheets in the space provided after each problem. Although no partial credit will be given on the multiple choice questions, any disputes about grades or grading will be settled by examining your written work on the question sheets.
5. Questions 8–10 are handwritten problems. **Write the solutions of the handwritten problems clearly and explain all steps.** You can use the back of the test pages for the scratch paper but it will not be looked for grading.
6. **NO CALCULATORS, BOOKS, NOTES, PHONES, OR CAMERAS ARE ALLOWED**.

   Turn off or put away all electronic devices.

1. Find the general solution of the differential equation

$$y^{(4)} + 6y'' + 9y = 0.$$

A. $y(t) = c_1 \sin(\sqrt{3}t) + c_2 \cos(\sqrt{3}t)$

B. $y(t) = c_1 \sin(3t) + c_2 \cos(3t) + c_3 t \sin(3t) + c_4 t \cos(3t)$

C. $y(t) = c_1 \sin(\sqrt{3}t) + c_2 \cos(\sqrt{3}t) + c_3 t \sin(\sqrt{3}t) + c_4 t \cos(\sqrt{3}t)$

D. $y(t) = c_1 e^{-3t} + c_2 t e^{-3t} + c_3 e^{3t} + c_4 t e^{3t}$

E. $y(t) = c_1 e^{-3t} + c_2 e^{3t} + c_3 e^{\sqrt{3}t} + c_4 e^{-\sqrt{3}t}$

2. A particular solution y_p of the differential equation:

$$y^{(3)} - 2y'' + y' = 1 + xe^x$$

is of the form

A. $y_p(x) = Ax + Bx^2e^x + Cx^3e^x$

B. $y_p(x) = Ax + Be^x + Cxe^x$

C. $y_p(x) = A + Bx^2e^x + Cx^3e^x$

D. $y_p(x) = A + Be^x + Cxe^x$

E. $y_p(x) = Ax + Bxe^x + Cx^2e^x$

3. Using the method of Variation of Parameters, find a particular solution $y_p(x)$ of the nonhomogeneous equation

$$y'' + P(x)y' + Q(x)y = 12x^2 \quad (x > 0)$$

given that $y_1(x) = 1$ and $y_2(x) = x^3$ are two linearly independent solutions to its corresponding homogeneous equation $y'' + P(x)y' + Q(x)y = 0$.

A. $y_p(x) = -6x^2$

B. $y_p(x) = 3x^4$

C. $y_p(x) = -\dfrac{4}{3}x^3 + 4x^3 \ln x$

D. $y_p(x) = 12x^2$

E. $y_p(x) = 2x^4$

4. Select the system of first-order differential equations that is equivalent to $x'' - 4x' + x^3 = 0$, given that $x_1 = x$ and $x_2 = x'$.

A. $x_1' = x_1 + x_2, \quad x_2' = -x_1^2 - 4x_2$

B. $x_1' = x_2, \quad x_2' = -x_1^3 + 4x_2$

C. $x_1' = x_2, \quad x_2' = 4x_1 + x_2^3$

D. $x_1' = x_2, \quad x_2' = x_1^3 - 4x_2$

E. $x_1' = 4x_2, \quad x_2' = x_1^3$

5. Let $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ be the solution of the initial value problem

$$\mathbf{x}' = \begin{bmatrix} 2 & -1 \\ 3 & 6 \end{bmatrix} \mathbf{x}; \quad \mathbf{x}(0) = \begin{bmatrix} 3 \\ -7 \end{bmatrix}.$$

Find $x_2(1)$.

A. $2e^3 + 4e^5$

B. $5e^2 - 6e^3$

C. $9e^3 - 7e^{-5}$

D. $-\cos 3 - 6 \sin 2$

E. $-e^3 - 6e^5$

6. Consider the system

$$\begin{cases} x' = \alpha x + y, \\ y' = -x. \end{cases}$$

For what value of α is the origin a center?

A. $\alpha = 0$

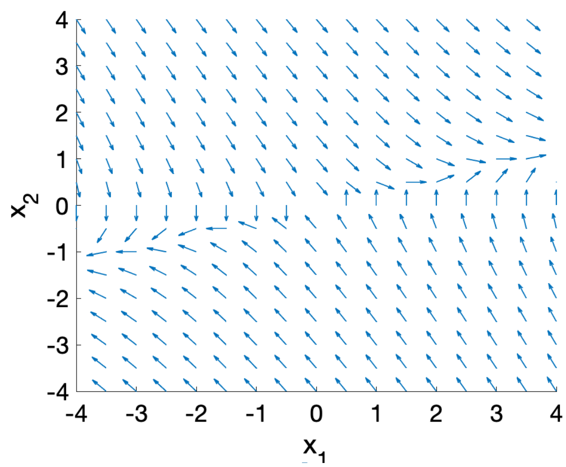
B. $\alpha = -2$

C. $\alpha < -2$

D. $-2 < \alpha < 0$

E. $0 < \alpha < 2$

7. Given the phase portrait of $\mathbf{x}' = A\mathbf{x}$ below, where $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, identify what type of eigenvalues the matrix A has.



A. purely imaginary eigenvalues

B. repeated eigenvalues

C. not enough information to decide

D. real eigenvalues of opposite sign

E. one zero and one negative, real eigenvalue

8. Find the solution $y(x)$ to the initial value problem

$$y'' + 2y' + y = 8e^x; \quad y(0) = 0,\ y'(0) = 0.$$

9. Suppose that the *undamped* and *free* mass-spring system with $m = 1$ and $k = 20$ is set in motion with $x(0) = 5$ and $x'(0) = 4$.

(a) Find the position function $x(t)$.

(b) The position function $x(t)$ can be written as $x(t) = C\cos(\omega t - \alpha)$. What are C and α?

10. Given that $\lambda = 4$ is a real eigenvalue with multiplicity 2 (meaning it is repeated), find a general solution of the system

$$\mathbf{x}' = \begin{bmatrix} 7 & -1 \\ 9 & 1 \end{bmatrix} \mathbf{x}.$$