MA26600 Exam 1

GREEN VERSION

NAME: ____________________

PUID (10 digits): ____________________

INSTRUCTOR: ____________________

SECTION/TIME: ____________________

1. Fill in your NAME, your PUID (10 digits), your INSTRUCTOR's name and SECTION number (or class meeting TIME) above. Please print legibly and use your name as it appears in the Purdue directory; please don't shorten or use a nickname.
2. Make sure you have all 8 pages of the exam book.
3. There are 10 questions, each worth 10 points.
4. Questions 1–7 are multiple-choice questions. Indicate your choice of an answer by **circling the letter** next to the choice like this:

   (D.) My choice of a correct answer.

   Show your work on the question sheets in the space provided after each problem. Although no partial credit will be given on the multiple choice questions, any disputes about grades or grading will be settled by examining your written work on the question sheets.
5. Questions 8–10 are handwritten problems. **Write the solutions of the handwritten problems clearly and explain all steps.** You can use the back of the test pages for the scratch paper but it will not be looked for grading.
6. **NO CALCULATORS, BOOKS, NOTES, PHONES, OR CAMERAS ARE ALLOWED**.

   Turn off or put away all electronic devices.

1. Find the general solution of the equation

$$(t^2+1)\frac{dy}{dt} + 4ty = 1.$$

A. $y = \dfrac{\frac{1}{2}t^2 + C}{t^2+1}$

B. $y = (t^2+1)(\tan^{-1}(t) + C)$

C. $y = e^{-4t}(\tan^{-1}(t) + C)$

D. $y = \dfrac{\frac{1}{3}t^3 + t + C}{(t^2+1)^2}$

E. $y = 2\ln(t^2+1) + C$

2. Solve the initial value problem

$$\frac{dy}{dx} = \frac{xy + y^2}{x^2}, \quad y(1) = \frac{1}{2}$$

for $x > 0$ using the substitution $v = y/x$.

A. $y = \dfrac{x}{2 + \ln x}$

B. $y = \dfrac{x}{2 - \ln x}$

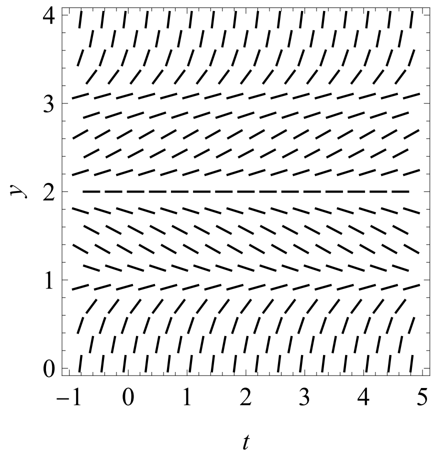
C. $y = \dfrac{1}{2 - \ln x}$

D. $y = \dfrac{x}{4 - 2x}$

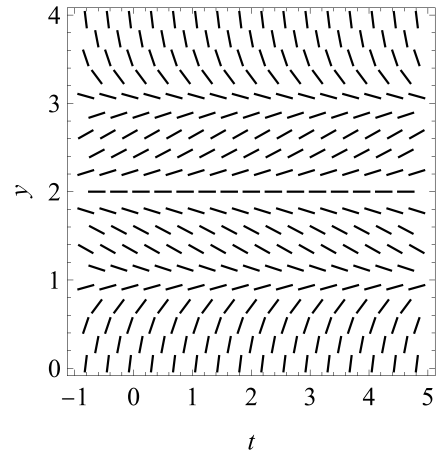
E. $y = \dfrac{x}{1 + x}$

3. Identify the direction field for the autonomous equation $\dfrac{dy}{dt} = (y-1)(y-2)(y-3)$.

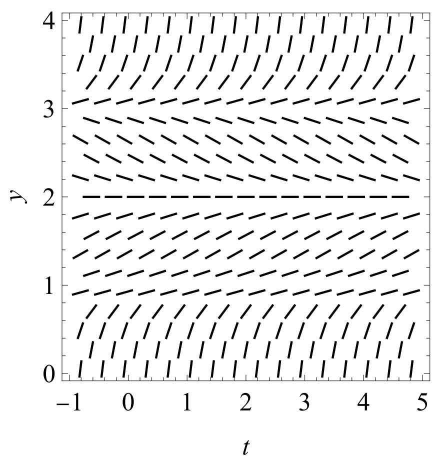
A.



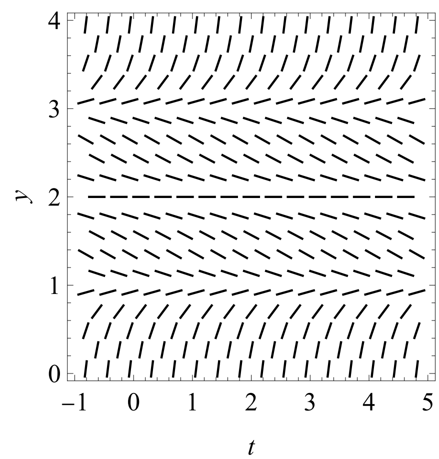
B.



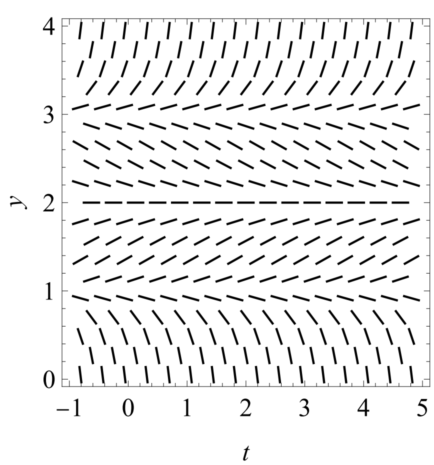
C.



D.



E.



4. Use Euler's method with two steps of size $h = 0.5$ to estimate $y(2)$, where $y(x)$ solves the initial value problem

$$y' = 2xy - 1 - \frac{y}{2x}, \quad y(1) = 2.$$

A. 4

B. 6.5

C. 7.5

D. 3

E. 7

5. Solve the initial value problem

$$y'' + y' - 12y = 0, \quad y(0) = 3, \; y'(0) = -4.$$

A. $y(t) = 3e^{-4t} - 4e^{3t}$

B. $y(t) = \frac{16}{7}\,e^{-3t} + \frac{5}{7}\,e^{4t}$

C. $y(t) = \frac{13}{7}\,e^{-4t} + \frac{8}{7}\,e^{3t}$

D. $y(t) = -\frac{5}{7}\,e^{-4t} - \frac{16}{7}\,e^{3t}$

E. $y(t) = \frac{11}{7}\,e^{-3t} + \frac{10}{7}\,e^{4t}$

6. A mass-spring-dashpot system with mass $m = 4$, damping constant $c = 4$, and spring constant $k = 17$ is set in free motion with initial conditions $x(0) = 0$ and $x'(0) = 2$, where $x(t)$ is the displacement from the equilibrium position at time t. Find $x(2)$.

A. $e^{-1} \sin(4)$

B. $e^{-1} \cos(4)$

C. $e^{-1} \cos(4) + e^{-1} \sin(4)$

D. $e^{-2} \cos(1) - e^{-2} \sin(1)$

E. $e^{-2} \sin(1)$

7. Which of the following differential equations has

$$y(t) = C_1 e^{4t} + C_2 t e^{4t} + C_3$$

as a general solution?

A. $y'' - 8y' + 16y = 0$

B. $y^{(3)} - 8y'' + 16y' + y = 0$

C. $y^{(3)} - 8y'' + 16y' = 0$

D. $y^{(3)} - 9y'' + 24y' - 16y = 0$

E. $y^{(3)} - 16y' = 0$

8. Find the value of the parameter α for which the equation is exact and then find an implicit solution of the initial value problem

$$(2xy^2 + x^2)dx + (\alpha x^2 y + 4y^3)dy = 0, \quad y(0) = 2.$$

9. A ball with mass 0.2 kg is thrown upward with initial velocity 49 m/s from the ground. There is a force due to air resistance of magnitude $|v|/25$ directed opposite to the velocity v (measured in m/s). How much time does it take for the ball to reach its maximum height? (Use that the gravitational acceleration $g = 9.8$ m/s^2.)

10. Consider the following differential equation:

$$\frac{dx}{dt} = (x+1)(3-x) + h,$$

where h is a parameter. Determine the number of critical points depending on the value of h.

(You don't need to solve the differential equation.)