MA26500 FINAL EXAM INSTRUCTIONS Fall 2018

GREEN - Test Version 01

NAME ______________________________ INSTRUCTOR ______________________________

1. You must use a **#2 pencil** on the mark–sense sheet (answer sheet).
2. On the mark–sense sheet, fill in the **instructor's** name (if you do not know, write down the class meeting time and location) and the **course number** which is **MA265**.
3. Fill in your **NAME** and blacken in the appropriate spaces.
4. Fill in the **SECTION Number** boxes with section number of your class (see below if you are not sure) and blacken in the appropriate spaces.

0011	TR	3:00PM	Caroline Madsen	0141	MWF	2:30PM	Ning Wei
0032	TR	1:30PM	Zhiguo Yang	0142	TR	7:30AM	Yating Wang
0033	TR	4:30PM	Baiying Liu	0143	MWF	12:30PM	Xu Wang
0041	TR	3:00PM	Andras Lorincz	0145	TR	4:30PM	Andras Lorincz
0051	TR	9:00AM	Yating Wang	0146	MWF	1:30PM	Ning Wei
0071	TR	3:00AM	Zhiguo Yang	0147	TR	4:30PM	Caroline Madsen
0111	MWF	2:30PM	Tao Luo	0148	MWF	3:30PM	Tao Luo
0121	MWF	12:30PM	Ying Zhang	0149	MWF	1:30PM	Brian Krummel
0122	MWF	9:30AM	Linquan Ma	0150	MWF	10:30AM	Ying Zhang
0131	MWF	11:30AM	Xu Wang	0151	MWF	11:30AM	Ying Zhang
0132	MWF	8:30AM	Linquan Ma	0152	MWF	2:30PM	Brian Krummel

5. Fill in the correct TEST/QUIZ NUMBER (**GREEN** is 01).
6. Fill in the 10-DIGIT PURDUE ID and blacken in the appropriate spaces.
7. Sign the mark–sense sheet.
8. Fill in your name and your instructor's name on the question sheets (above).
9. There are 25 questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25 in the answer sheet. Do all your work on the question sheets, in addition, also **CIRCLE** your answer choice for each problem on the question sheets in case your scantron is lost. **Turn in both the mark-sense sheets and the question sheets to your instructor when you are finished**.
10. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.
11. NO CALCULATORS, BOOKS, NOTES, PHONES OR CAMERAS ARE ALLOWED on this exam. Turn off or put away all electronic devices. Use the back of the test pages for scrap paper.

1. Consider the system of linear equations

$$\begin{aligned} x + 2y + \ \ 3z &= 1 \\ 3x + 5y + \ \ 4z &= a \\ 2x + 3y + a^2 z &= 0. \end{aligned}$$

For which value of a is the system inconsistent?

A. $a = -1$

B. $a = 2$

C. $a = 1$

D. $a = -2$

E. $a = 3$

2. Consider the equation $A\mathbf{x} = \mathbf{b}$ where A is a 3×2 matrix and $\mathbf{b}$ is in $\mathbb{R}^3$. Which of the following statements is true for every matrix A?

A. The equation $A\mathbf{x} = \mathbf{b}$ is inconsistent for every $\mathbf{b}$ in $\mathbb{R}^3$.

B. Whenever the equation $A\mathbf{x} = \mathbf{b}$ is consistent, it has exactly one solution $\mathbf{x}$.

C. Whenever the equation $A\mathbf{x} = \mathbf{b}$ is consistent, it has infinitely many solutions $\mathbf{x}$.

D. The equation $A\mathbf{x} = \mathbf{b}$ is inconsistent for at least one $\mathbf{b}$ in $\mathbb{R}^3$.

E. If the columns of A are a scalar multiple of one another, then the equation $A\mathbf{x} = \mathbf{b}$ has exactly one solution.

3. Suppose $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ a & 5 & 1 \end{bmatrix}$. What is a?

A. 1

B. 2

C. 3

D. 4

E. 0

4. Which of the following subsets of $\mathbb{R}^3$ is linearly independent?

A. $\left\{ \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 8 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 18 \\ 12 \\ 6 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 8 \\ 0 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 0 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 8 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix} \right\}$

5. Let $A^T = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 \end{bmatrix}$. Find the $(3,1)$ entry of $A^T A$.

A. 74

B. 86

C. 38

D. 108

E. 62

6. Find a basis for the null space of $A = \begin{bmatrix} -3 & 6 & -1 & -4 \\ 1 & -2 & 2 & 3 \\ 2 & -4 & 5 & 7 \end{bmatrix}$.

A. $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \\ 1 \end{bmatrix} \right\}$.

B. $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$.

C. $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \end{bmatrix} \right\}$.

D. $\left\{ \begin{bmatrix} 3 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$.

E. $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -2 \\ 1 \end{bmatrix} \right\}$.

7. Compute the determinant of the matrix A where

$$A = \begin{bmatrix} 0 & 3 & 4 & 0 & 0 \\ 0 & 3 & 2 & 0 & 5 \\ 1 & 2 & 0 & 2 & 3 \\ 0 & 0 & -3 & 0 & 0 \\ 3 & 2 & 0 & 2 & 4 \end{bmatrix}.$$

A. -180

B. -60

C. 60

D. 180

E. 0

8. If the determinant of

$$\begin{bmatrix} 2c_1 & -2c_2 & 2c_3 \\ a_1 + 6c_1 & -a_2 - 6c_2 & a_3 + 6c_3 \\ -b_1 & b_2 & -b_3 \end{bmatrix}$$

is -16, what is the determinant of

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}?$$

A. 32

B. -32

C. -16

D. 8

E. -8

9. Let

$$A = \begin{bmatrix} 3 & 5 & 8 \\ -2 & -2 & -9 \\ 2 & 4 & 5 \end{bmatrix}.$$

Then the $(1,2)$-entry of A^{-1} is:

A. 6

B. $-\frac{7}{6}$

C. $\frac{7}{6}$

D. $-\frac{4}{3}$

E. $\frac{4}{3}$

10. Which of the following sets of vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ are subspaces of $\mathbb{R}^3$?

(i) The set of all vectors satisfying the condition $xyz = 0$.

(ii) The set of all vectors satisfying the condition $3x + 5y + z = 0$.

(iii) The set of all vectors satisfying the condition $x^2 + y^2 + z^2 = 0$.

(iv) The set of all vectors satisfying the condition $x + y + z = 1$.

(v) The set of all vectors satisfying the condition $x = y$.

A. (i) and (ii) only

B. (ii) and (iii) only

C. (ii), (iii) and (v) only

D. (i), (ii) and (iii) only

E. (i), (ii) and (v) only

11. Let $A = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{v}_4 & \mathbf{v}_5 \end{bmatrix}$ be a 4×5 matrix. Assume that $\mathbf{v}_3 = \mathbf{v}_1 + \mathbf{v}_2$ and $\mathbf{v}_4 = 2\mathbf{v}_1 - \mathbf{v}_2$. What can you say about the rank and nullity of A?

A. $\operatorname{rank} A \leq 3$ and $\operatorname{nullity} A \geq 2$

B. $\operatorname{rank} A \geq 2$ and $\operatorname{nullity} A \leq 3$

C. $\operatorname{rank} A \geq 3$ and $\operatorname{nullity} A \leq 2$

D. $\operatorname{rank} A \leq 2$ and $\operatorname{nullity} A \geq 2$

E. $\operatorname{rank} A \geq 2$ and $\operatorname{nullity} A \leq 2$

12. Which of the following sets of vectors is linearly dependent?

A. $\left\{ \begin{bmatrix} -4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \end{bmatrix} \right\}$ in $\mathbb{R}^2$

B. $\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 0 \end{bmatrix} \right\}$ in $\mathbb{R}^3$

C. $\left\{ \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 5 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix} \right\}$ in $\mathbb{R}^3$

D. $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}, \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix} \right\}$ in $M_{2\times 2}$, the vector space of all 2×2 matrices

E. $\{t^2 - 2t, t^2 + 2t, t^2 + 2t + 5\}$ in $\mathbb{P}_2(t)$, the vector space of all the polynomials in t with degree ≤ 2

13. Let $A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 3 & 5 & -2 & -2 \\ -2 & 3 & 3 & 5 \end{bmatrix}$. Which of the following values is a *multiple* eigenvalue of A?

A. -1

B. -2

C. 1

D. 2

E. 4

14. Let T be the linear transformation whose standard matrix is $\begin{bmatrix} 1 & -4 & 8 & 1 \\ 0 & 2 & -1 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix}$. Which of the following statements are true?

(i) T maps $\mathbb{R}^3$ onto $\mathbb{R}^4$
(ii) T maps $\mathbb{R}^4$ onto $\mathbb{R}^3$
(iii) T is onto
(iv) T is one-to-one

A. (i) and (iii) only

B. (i) and (iv) only

C. (ii) and (iv) only

D. (ii) and (iii) only

E. (ii), (iii) and (iv) only

15. Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$. Then A^6 is:

A. $\begin{bmatrix} 1 & 1 \\ 0 & 64 \end{bmatrix}$

B. $\begin{bmatrix} 1 & -63 \\ 0 & 64 \end{bmatrix}$

C. $\begin{bmatrix} -1 & -63 \\ 0 & -64 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 63 \\ 0 & 64 \end{bmatrix}$

E. $\begin{bmatrix} -1 & 63 \\ 0 & -64 \end{bmatrix}$

16. Which of the following matrices are diagonalizable?

(i) $\begin{bmatrix} 1 & 4 \\ 1 & -2 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 1 & -2 \\ 0 & 0 & 4 \\ 0 & 0 & 6 \end{bmatrix}$ (iv) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 2 \\ 3 & 2 & 2 \end{bmatrix}$

A. (i) and (iii) only

B. (iii) and (iv) only

C. (ii) and (iii) only

D. (i), (ii) and (iv) only

E. (i) , (ii), (iii) and (iv)

17. Consider the dynamical system $\mathbf{x}' = A\mathbf{x}$, where $A = \begin{bmatrix} 3 & 1 \\ -5 & -3 \end{bmatrix}$. Then the origin is

A. an attractor

B. a repeller

C. a saddle point

D. a spiral point

E. none of the above

18. Which one of the following is the solution to the differential equation

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

with initial condition $\begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$?

A. $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} e^{5t} \\ e^{5t} \end{bmatrix} + \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$

B. $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 3e^{5t} \\ 3e^{5t} \end{bmatrix} + \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix}$

C. $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 3e^{5t} \\ 3e^{5t} \end{bmatrix} + \begin{bmatrix} -e^{-t} \\ e^{-t} \end{bmatrix}$

D. $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} e^{5t} \\ e^{-t} \end{bmatrix} + \begin{bmatrix} e^{5t} \\ -e^{-t} \end{bmatrix}$

E. $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 3e^{5t} \\ e^{-t} \end{bmatrix} + \begin{bmatrix} 3e^{5t} \\ -e^{-t} \end{bmatrix}$

19. Let $A = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$. Then $\begin{bmatrix} -\mathbf{i} \\ 1 \end{bmatrix}$

A. is an eigenvector corresponding to eigenvalue 0.

B. is an eigenvector corresponding to eigenvalue $4\mathbf{i}$.

C. is an eigenvector corresponding to eigenvalue $-4\mathbf{i}$.

D. is an eigenvector corresponding to eigenvalue -4.

E. is an eigenvector corresponding to eigenvalue 4.

20. Let $W = \text{Span}\{\mathbf{u}_1, \mathbf{u}_2\}$ where $\mathbf{u}_1 = \begin{bmatrix} 2 \\ 5 \\ -1 \end{bmatrix}$ and $\mathbf{u}_2 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$. Let $\mathbf{y} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$, find the orthogonal projection of $\mathbf{y}$ onto W.

A. $\begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}$

B. $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}$

D. $\begin{bmatrix} -2 \\ -2 \\ -1 \end{bmatrix}$

E. $\begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$

21. For the matrix $A = \begin{bmatrix} 5 & 1 \\ 0 & 5 \end{bmatrix}$, which of the following statements is true?

A. A is both invertible and diagonalizable.

B. A is invertible but not diagonalizable.

C. A is diagonalizable but not invertible.

D. A is neither invertible nor diagonalizable.

E. There is not enough information to determine whether A is diagonalizable or invertible.

22. Find a least-squares solution for $A\mathbf{x} = \mathbf{b}$ where $A = \begin{bmatrix} -1 & 2 \\ 2 & -3 \\ -1 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.

A. $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

B. $\begin{bmatrix} -6 \\ 11 \end{bmatrix}$

C. $\begin{bmatrix} -7 \\ 22 \end{bmatrix}$

D. $\begin{bmatrix} -3 \\ -1 \end{bmatrix}$

E. $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

23. Let V be the vector space $C[-1,1]$ (vector space of all continuous functions on $[-1,1]$) with the inner product given by $\langle f, g\rangle = \int_{-1}^{1} f(t)g(t)dt$. Find an orthogonal basis for the subspace spanned by the polynomials 1, $t+1$, and t^2.

A. $\{1,\, 3t-1,\, 11t^2\}$

B. $\{1,\, t,\, 3t^2-1\}$

C. $\{1,\, 3t-1,\, 3t^2-2\}$

D. $\{1,\, t,\, 3t^2-2\}$

E. $\{1,\, 2t-1,\, 11t^2\}$

24. Which of the following statements are/is true?

(i) An $n \times n$ matrix that is orthogonally diagonalized must be symmetric.

(ii) An $n \times n$ symmetric matrix has n distinct real eigenvalues.

(iii) There are symmetric matrices that are not orthogonally diagonalizable.

(iv) If $B = PDP^T$, where $P^T = P^{-1}$ and D is a diagonal matrix, then B is a symmetric matrix.

A. (iii) only

B. (i) and (iv) only

C. (i) and (iii) only

D. (ii) and (iv) only

E. (i), (ii), (iii) and (iv)

25. Let $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, which are linearly independent. If we apply the Gram-Schmidt process to this basis of $W = \text{Span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ to obtain an orthonormal basis for W, we obtain:

A. $\frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{12}}\begin{bmatrix} -3 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{6}}\begin{bmatrix} 0 \\ -2 \\ -1 \\ 1 \end{bmatrix}$

B. $\frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{12}}\begin{bmatrix} -3 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{6}}\begin{bmatrix} 0 \\ -2 \\ 1 \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

D. $\frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{12}}\begin{bmatrix} -3 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{6}}\begin{bmatrix} 0 \\ -2 \\ 1 \\ -1 \end{bmatrix}$

E. $\frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{12}}\begin{bmatrix} -3 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{6}}\begin{bmatrix} 0 \\ -2 \\ 1 \\ 1 \end{bmatrix}$