MA26500 FINAL EXAM INSTRUCTIONS Fall 2017

GREEN - Test Version 01

NAME ______________________ INSTRUCTOR ______________________

1. You must use a **#2 pencil** on the mark–sense sheet (answer sheet).
2. On the mark–sense sheet, fill in the **instructor's** name (if you do not know, write down the class meeting time and location) and the **course number** which is **MA265**.
3. Fill in your **NAME** and blacken in the appropriate spaces.
4. Fill in the **SECTION Number** boxes with section number of your class (see below if you are not sure) and blacken in the appropriate spaces.

0011	MWF	12:30PM	Yong Suk Moon	0033	MWF	2:30PM	Anantharam Raghuram
0032	MWF	11:30AM	Yong Suk Moon	0142	TR	9:00AM	Ying Chen
0041	MWF	2:30PM	Marius Dadarlat	0143	TR	1:30PM	Zhiguo Yang
0051	MWF	1:30PM	Robert Zink	0145	MWF	9:30AM	Meng-Che "Turbo" Ho
0071	TR	10:30AM	Ying Chen	0146	MWF	3:30PM	Anantharam Raghuram
0111	MWF	3:30PM	Fan Gao	0147	TR	3:00PM	Andras Lorincz
0121	TR	10:30AM	Peter Patzt	0148	MWF	4:30PM	Fan Gao
0122	TR	1:30PM	Andras Lorincz	0149	MWF	8:30AM	Meng-Che "Turbo" Ho
0131	TR	3:00PM	Zhiguo Yang	0150	TR	12:00PM	Peter Patzt
0132	MWF	11:30AM	Edray Goins	0151	MWF	3:30PM	Moongyu Park
0141	MWF	2:30PM	Moongyu Park				

5. Fill in the correct TEST/QUIZ NUMBER (**GREEN** is 01).
6. Fill in the 10-DIGIT PURDUE ID and blacken in the appropriate spaces.
7. Sign the mark–sense sheet.
8. Fill in your name and your instructor's name on the question sheets (above).
9. There are 25 questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25 in the answer sheet. Do all your work on the question sheets, in addition, also **CIRCLE** your answer choice for each problem on the question sheets in case your scantron is lost. **Turn in both the mark-sense sheets and the question sheets to your instructor when you are finished**.
10. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.
11. NO CALCULATORS, BOOKS, NOTES, PHONES OR CAMERAS ARE ALLOWED on this exam. Turn off or put away all electronic devices. Use the back of the test pages for scrap paper.

1. Let $A = \begin{bmatrix} 3 & 2 \\ 0 & -1 \end{bmatrix}$. Which one of the following statements is FALSE?

A. For arbitrary 2×2 matrices B, C, if $BA = BC$ then $A = C$.

B. A^T is invertible.

C. For arbitrary 2×2 matrices B, C, if $AB = AC$ then $B = C$.

D. $AA^{-1} = A^{-1}A$.

E. A^{-1} is an upper triangular matrix.

2. For a real number a, consider the system of equations

$$\begin{array}{rcrcrcr} x & + & y & + & z & = & 2 \\ 2x & + & 3y & + & 3z & = & 4 \\ 2x & + & 3y & + & (a^2-1)z & = & a+2 \end{array}$$

Which of the following statements is true?

A. If $a = 3$ then the system is inconsistent.

B. If $a = 1$ then the system has infinitely many solutions.

C. If $a = -1$ then the system has at least two distinct solutions.

D. If $a = 2$ then the system has a unique solution.

E. If $a = -2$ then the system is inconsistent.

3. Let A be an $n \times n$ nonsingular matrix. Which of the following statements must be true?

(i) $\det(A) \neq 0$.

(ii) $\det(A) = 0$.

(iii) $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions.

(iv) $A\mathbf{x} = \mathbf{b}$ has a unique solution for every n-vector $\mathbf{b}$.

(v) A must be row equivalent to the identity matrix.

A. (i) and (iii) only.

B. (ii) and (iv) only.

C. (i), (iv) and (v).

D. (ii), (iv) and (v).

E. (i), (iii) and (v).

4. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}.$$

Then the $(2,3)$-entry of A^{-1} is:

A. -1

B. 0

C. 1

D. 2

E. -2

5. The determinant of the matrix $\begin{bmatrix} -1 & 2 & 0 & 1 \\ 3 & 4 & 2 & 0 \\ -8 & 3 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{bmatrix}$ is

A. 0

B. -10

C. 10

D. -24

E. 24

6. Let $ad - bc = 2$ and $c = 2a - 4$. Find the value of y determined by the linear system

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

A. 1

B. 2

C. 3

D. 4

E. 5

7. If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1+i \end{bmatrix}$ has inverse $A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $a+b+c+d$ is equal to:

A. 1

B. $2+i$

C. $2-2i$

D. $4+i$

E. $4-i$

8. Consider the following subsets of the vector space $\mathbb{R}^4$. Which are subspaces of $\mathbb{R}^4$?

(i) The set of all elements of the form $\begin{bmatrix} 2x \\ 3x \\ -x \\ 5x \end{bmatrix}$.

(ii) The set of all elements of the form $\begin{bmatrix} x \\ x \\ -3x \\ 1 \end{bmatrix}$.

(iii) The set of all solutions for the homogeneous system $\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 4 & 2 & 4 \\ -1 & 0 & 3 & -5 \\ 1 & 0 & 7 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

A. (i) only

B. (iii) only

C. (i), (ii) only

D. (i), (iii) only

E. (i), (ii), (iii)

9. Consider the vector space P_2 of polynomials with real coefficients of degree at most 2. Which of the following subsets span P_2?

A. $\{t^2,\ t+1\}$

B. $\{t^2,\ 3t,\ t^2+t\}$

C. $\{t^2+1,\ t^2+2, 4\}$

D. $\{t^2+t+1,\ t-1,\ 2t^2+3t+1\}$

E. $\{t^2+4,\ t^2+2t,\ t+3\}$

10. Which of the following subsets of $\mathbb{R}^3$ is linearly independent?

A. $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 10 \\ 15 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 5 \\ 14 \\ 13 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \right\}$

11. Determine all the values of t such that the following vectors form a basis of $\mathbb{R}^3$:

$$\begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ t \\ 2 \end{bmatrix}, \begin{bmatrix} t \\ -3 \\ 2 \end{bmatrix}.$$

A. $t \neq -2, 1$

B. $t \neq -1, 2$

C. $t = 1, 2$

D. $t = -1, 2$

E. $t \neq -2, -1$

12. Let A be a 4×5 matrix, and let B denote the reduced row echelon form of A. Which of the following statements is TRUE?

A. If the columns 1,3,5 of B have leading 1's then the columns 1,3,5 of A span $\mathbb{R}^4$.

B. If the number of non-zero rows of B equals 3, then the number of non-zero columns of B equals 3.

C. If B has three leading 1's, then the nullity of A is 3.

D. If the number of non-zero rows of B equals 2, then one can pick 2 columns of A that form a basis of the column space of A.

E. None of the above.

13. Which of the following statements are always true?

(i) If A is an $m \times n$ matrix, then the solution space $\{\mathbf{x} | A\mathbf{x} = \mathbf{0}\}$ is a subspace of R^n

(ii) A is nonsingular if and only if A is a product of elementary matrices.

(iii) If A is an $n \times n$ matrix and k is a scalar, then $\det(kA) = k \det(A)$.

(iv) Let A be an $m \times n$ matrix, then the row space of A is orthogonal to the null space of A.

A. (i) and (ii) only

B. (i), (ii), and (iii)

C. (ii), (iii), and (iv)

D. (i), (ii), and (iv)

E. All of the above

14. Find the sum of all the eigenvalues of $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$.

A. 10

B. 9

C. 6

D. 4

E. 2

15. Find the nullity of the matrix A, where

$$A = \begin{bmatrix} 1 & -2 & 0 & 3 & -4 \\ 3 & 2 & 8 & 1 & 4 \\ -1 & 2 & 0 & 4 & -3 \\ 1 & 5 & 7 & 6 & 0 \end{bmatrix}.$$

A. 0

B. 1

C. 2

D. 3

E. 4

16. Suppose that $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ is orthogonal to both $\begin{bmatrix} 3 \\ 4 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 2 \\ -3 \end{bmatrix}$ and a, b, c are not all 0. Which of the following is true?

A. $a/b = -5/7$

B. $b/c = -1$

C. $c/a = -1/6$

D. $a/c = 4/5$

E. There is no solution.

17. $S = \left\{ \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ -4 \\ 3 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^3$. If we apply the Gram-Schmidt process to this basis to obtain an orthonormal basis for $\mathbb{R}^3$, we obtain:

A. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

B. $\left\{ \frac{1}{\sqrt{5}} \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{45}} \begin{bmatrix} 2 \\ -5 \\ 4 \end{bmatrix}, \frac{1}{\sqrt{27}} \begin{bmatrix} -1 \\ -1 \\ 5 \end{bmatrix} \right\}$

C. $\left\{ \frac{1}{\sqrt{5}} \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \right\}$

D. $\left\{ \frac{1}{\sqrt{5}} \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \frac{1}{\sqrt{10}} \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} \right\}$

E. $\left\{ \frac{1}{\sqrt{5}} \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{45}} \begin{bmatrix} 2 \\ -5 \\ 4 \end{bmatrix}, \frac{1}{3} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \right\}$

18. Consider $\mathbb{R}^2$ as an Euclidean space with respect to the standard inner product. Let V be the line spanned by $\begin{bmatrix} 5 \\ 8 \end{bmatrix}$. Which is the equation of the line that represents the orthogonal complement $V^{\perp}$ of V?

A. $8\,x + 5\,y = 0$

B. $-8\,x + 5\,y = 0$

C. $5\,x + 8\,y = 0$

D. $-5\,x + 8\,y = 0$

E. None of the above

19. Consider the linear system

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}.$$

Which of the following is the least squares solution $\widehat{\mathbf{x}} = \begin{bmatrix} a \\ b \end{bmatrix}$?

A. $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

B. $\begin{bmatrix} 24 \\ 6 \end{bmatrix}$

C. $\begin{bmatrix} 18 \\ 38 \end{bmatrix}$

D. $\begin{bmatrix} 282 \\ 640 \end{bmatrix}$

E. There is no solution

20. Let $V = \mathbb{R}^1$. Which of the following is a linear operator $L : V \to V$?

(i) $L(x) = 3\,x + 7$

(ii) $L(x) = 5\,x^2$

(iii) $L(x) = 2\,x$

A. (ii) only

B. (iii) only

C. (i) and (iii)

D. (i), (ii), and (iii)

E. None of the above

21. Let $A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$. Then $\begin{bmatrix} -i \\ 1 \end{bmatrix}$

A. is an eigenvector with eigenvalue 2

B. is an eigenvector with eigenvalue $2i$

C. is an eigenvector with eigenvalue -2

D. is an eigenvector with eigenvalue $-2i$

E. is an eigenvector with eigenvalue 0

22. Which of the following matrices is *not* diagonalizable?

A. $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$

C. $\begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 2 \\ 2 & 2 \end{bmatrix}$

E. $\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$

23. Let $x(t)$ and $y(t)$ be the unique solutions of the initial value problem:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 5 & -4 \\ -4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}, \quad \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}.$$

Then $x(1)$ is equal to:

A. $e^9 - e^{-9}$

B. $e^9 + 4e$

C. $e^3 - 2e^{-3}$

D. $e^3 + 2e^{-3}$

E. $3e^9 + e$

24. Consider the dynamical system $\mathbf{x}' = A\mathbf{x}$, where $A = \begin{bmatrix} 1 & 4 \\ 1 & -2 \end{bmatrix}$. The origin is

A. a saddle point

B. stable

C. unstable

D. marginally stable

E. none of the above

25. Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 2 & 1 \\ 2 & 4 & 0 & 2 & -2 \\ 0 & 0 & 0 & 2 & 4 \end{bmatrix}$, which of the following is a basis of the column space of (A)?

A. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 0 \\ 0 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 2 \\ 2 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ -2 \\ 2 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 2 \\ 2 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ -2 \\ 2 \end{bmatrix} \right\}$