MA 265, Fall 2023, Midterm II (GREEN)

INSTRUCTIONS:

1. **Write your answers of the seven multiple choice questions into the table on the last page. Show all your work on the questions and you may use the back of the test pages as scratch paper if needed.**

2. After you have finished the exam, hand in your test booklet to your instructor.

101	MWF	12:30PM	Ying Zhang	600	MWF	10:30AM	Farrah Yhee
102	MWF	11:30AM	Ying Zhang	601	MWF	11:30AM	Farrah Yhee
153	MWF	1:30PM	Ying Zhang	650	MWF	11:30AM	Yiran Wang
154	MWF	11:30AM	Takumi Murayama	651	MWF	12:30PM	Yiran Wang
205	MWF	11:30AM	Jing Wang	701	TR	12:00PM	Raechel Polak
206	MWF	2:30PM	Sam Nariman	702	TR	4:30PM	Vaibhav Pandey
357	MWF	8:30AM	Jonathon Peterson	703	MWF	10:30AM	Takumi Murayama
410	MWF	11:30AM	Siamak Yassemi	704	MWF	8:30AM	Anurag Sahay
451	MWF	1:30PM	Siamak Yassemi	705	MWF	1:30PM	Yi Wang
501	MWF	1:30PM	Yilong Zhang	706	MWF	12:30PM	Yi Wang
502	MWF	10:30AM	Yilong Zhang	707	TR	3:00PM	Vaibhav Pandey

3. NO CALCULATORS, BOOKS, NOTES, PHONES OR CAMERAS ARE ALLOWED on this exam. Turn off or put away all electronic devices.

4. When time is called, all students must put down their writing instruments immediately. You may remain in your seat while your instructor will collect the exam booklets.

5. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above instructions regarding academic dishonesty:

STUDENT NAME ______________________________

STUDENT SIGNATURE ______________________________

STUDENT PUID ______________________________

SECTION NUMBER ______________________________

1. (10 points) Which of the following sets of vectors is linearly dependent?

A. $\{1, t, t^2\}$

B. $\{1 + t, t, t^2 + 1\}$

C. $\{1 + t, t - 1, t^2 - 1\}$

D. $\{1 + t + t^2, -2 + t^2, -2t^2\}$

E. $\{1 - t, t - t^2, -3 + 3t^2\}$

2. (10 points) Compute the dimension of the subspace

$$H = \text{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 4 \\ 8 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 4 \\ 12 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \\ -4 \end{bmatrix} \right\}.$$

A. 0

B. 1

C. 2

D. 3

E. 4

3. (10 points) Consider the system of differential equations

$$x'(t) = 7x(t) - 4y(t)$$

$$y'(t) = 8x(t) - y(t)$$

Then the origin is

A. a saddle point

B. an attractor

C. a repeller

D. a spiral point with the trajectory spiraling inward

E. a spiral point with the trajectory spiraling outward

4. (10 points) Let A be a 5×9 matrix with $\text{rank}(A) = 4$. Which of the following statements must be **TRUE**?

(i) $\text{rank}(A^T) = 5$

(ii) $\dim(\text{row}(A)) = 4$

(iii) $\dim(\text{nul}(A)) = 1$

(iv) $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions for every $\mathbf{b}$ in $\mathbb{R}^5$.

(v) The columns of A are linearly dependent.

A. (ii) and (v) only

B. (i) and (v) only

C. (iii) and (iv) only

D. (ii), (iv), and (v) only

E. (i), (iv), and (v) only

5. (10 points) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation. Suppose $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ is a basis for $\mathbb{R}^3$ such that $T(\mathbf{b}_1) = \mathbf{b}_1 + 2\mathbf{b}_2 + 3\mathbf{b}_3$, $T(\mathbf{b}_2) = \mathbf{b}_2 + \mathbf{b}_3$ and $T(\mathbf{b}_3) = \mathbf{b}_1 - \mathbf{b}_2 + 2\mathbf{b}_3$. Let A be the matrix for T relative to $\mathcal{B}$, find $\det A$.

A. 1

B. 2

C. 3

D. 4

E. 5

6. (10 points) Let A be an $n \times n$ matrix with real entries and B be an echelon form of A. Which of the following statements is not always true?

A. $\text{Row}(A) = \text{Row}(B)$.

B. 0 is an eigenvalue of A if and only if 0 is an eigenvalue of B.

C. 0 is not an eigenvalue of A if and only if 0 is not an eigenvalue of B.

D. A and B have the same eigenvalues.

E. If $\mathbf{v}$ is a complex eigenvector of A corresponding to an eigenvalue λ of A, then $\bar{\mathbf{v}}$ is an eigenvector of A corresponding to $\bar{\lambda}$.

7. (10 points) Let $A = \begin{bmatrix} -5 & 6 \\ -3 & 4 \end{bmatrix}$. What is A^{2023}?

A. $\begin{bmatrix} 3 \times 2^{2023} - 2 & 3 - 3 \times 2^{2023} \\ 2 \times 2^{2023} - 2 & 3 - 2 \times 2^{2023} \end{bmatrix}$

B. $\begin{bmatrix} 3 \times 2^{2023} + 2 & 3 + 3 \times 2^{2023} \\ 2 \times 2^{2023} - 2 & 3 - 2 \times 2^{2023} \end{bmatrix}$

C. $\begin{bmatrix} -2 \times 2^{2023} - 1 & 2 \times 2^{2023} + 2 \\ -2^{2023} - 1 & 2^{2023} + 2 \end{bmatrix}$

D. $\begin{bmatrix} -2 \times 2^{2023} - 1 & 2 \times 2^{2023} - 2 \\ -2^{2023} - 1 & 2^{2023} - 2 \end{bmatrix}$

E. $\begin{bmatrix} 4096 & -3072 \\ 2048 & -1536 \end{bmatrix}$

8. Let $\mathbb{P}_2 = \{p(t) = a + bt + ct^2 \ : \ a, b, \text{and } c \text{ are any real numbers}\}$ be the vector space consisting of polynomials with degree at most two and the zero polynomial. Consider the linear transformation:

$$T : \mathbb{P}_2 \to \mathbb{R}^3$$
$$T(p(t)) = \begin{bmatrix} p(-1) \\ p(1) \\ 0 \end{bmatrix}.$$

(3 points) (1) Find all the polynomials $p(t)$ in $\mathbb{P}_2$ such that $T(p(t)) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

(3 points) (2) Determine a basis for the kernel of T.

(4 points) (3) Determine a basis for the range of T.

9. (4 points) (1) Find all the eigenvalues of matrix $A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ 1 & -1 & 2 \end{bmatrix}$.

(6 points) (2) Find a basis for the eigenspace corresponding to each of the eigenvalues of matrix A.

10. Given that vectors $\mathbf{v}_1 = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ are the eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}.$$

(2 points) (1) Find their corresponding eigenvalues λ_1 and λ_2.

(2 points) (2) Find a general solution to the system of differential equations

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}.$$

(6 points) (3) Let $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ be a particular solution to the initial value problem

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, \quad \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}.$$

Find $x(1) + y(1)$.

Please write your answers of the 7 multiple choice questions in the following table.

Question	Answer
1. (10 points)	
2. (10 points)	
3. (10 points)	
4. (10 points)	
5. (10 points)	
6. (10 points)	
7. (10 points)	

Total Points: ________________________