MA 265, Fall 2022, Midterm II (GREEN)

INSTRUCTIONS:

1. **Write your answers of the seven multiple choice questions into the table on the last page. Show all your work on the questions and you may use the back of the test pages as scratch paper if needed.**

2. After you have finished the exam, hand in your test booklet to your instructor.

101	MWF	10:30AM	Ying Zhang
102	MWF	9:30AM	Ying Zhang
153	MWF	11:30AM	Ying Zhang
154	MWF	11:30AM	Daniel Tuan-Dan Le
205	TR	1:30PM	Oleksandr Tsymbaliuk
206	TR	3:00PM	Oleksandr Tsymbaliuk
357	MWF	1:30PM	Yiran Wang
410	TR	1:30PM	Arun Debray
451	TR	10:30AM	Arun Debray
501	TR	12:00PM	Vaibhav Pandey
502	MWF	10:30AM	Ayan Maiti
600	MWF	1:30PM	Seongjun Choi
601	MWF	1:30PM	Ayan Maiti
650	MWF	10:30AM	Yevgeniya Tarasova
651	MWF	9:30AM	Yevgeniya Tarasova
701	MWF	3:30PM	Seongjun Choi
702	MWF	11:30AM	Yiran Wang
703	MWF	12:30PM	Ke Wu
704	MWF	1:30PM	Ke Wu
705	MWF	12:30PM	Seongjun Choi
706	TR	1:30PM	Vaibhav Pandey
707	MWF	3:30PM	Siamak Yassemi
708	MWF	2:30PM	Siamak Yassemi

3. NO CALCULATORS, BOOKS, NOTES, PHONES OR CAMERAS ARE ALLOWED on this exam. Turn off or put away all electronic devices.

4. When time is called, all students must put down their writing instruments immediately. You may remain in your seat while your instructor will collect the exam booklets.

5. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above instructions regarding academic dishonesty:

STUDENT NAME ____________________

STUDENT SIGNATURE ____________________

STUDENT PUID ____________________

SECTION NUMBER ____________________

1. (10 points) Let

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 & -1 \\ 1 & 2 & 4 & -2 & -1 \\ 2 & 3 & 7 & -3 & -2 \end{bmatrix}$$

Let a be the rank of A and b be the nullity of A, find $5b - 3a$.

A. 25

B. 17

C. 9

D. 1

E. 0

2. (10 points) Let $\mathbf{u} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 1 \\ -1 \\ c \end{bmatrix}$ where c is a real number. The set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is a basis for $\mathbb{R}^3$ provided that c is not equal

A. -2

B. 2

C. -3

D. 3

E. -1

3. (10 points) Which of the following statements is always TRUE?

A. If $A\mathbf{x} = \lambda\mathbf{x}$ for some vector $\mathbf{x}$, then λ is an eigenvalue of A.

B. If $\mathbf{v}$ is an eigenvector corresponding to eigenvalue 2, then $-\mathbf{v}$ is an eigenvector corresponding to eigenvalue -2.

C. If B is invertible, then matrices A and $B^{-1}AB$ could have different sets of eigenvalues.

D. If λ is an eigenvalue of matrix A, then λ^2 is an eigenvalue of matrix A^2.

E. If -5 is an eigenvalue of matrix B, then matrix $B - 5I$ is not invertible.

4. (10 points) Let $\mathbb{P}_3$ be the vector space of all polynomials of degree at most 3. Which of the following subsets are subspaces of $\mathbb{P}_3$?

(i) A set of polynomials in $\mathbb{P}_3$ satisfying $p(0) = p(1)$.

(ii) A set of polynomials in $\mathbb{P}_3$ satisfying $p(0)p(1) = 0$.

(iii) A set of polynomials in $\mathbb{P}_3$ with integer coefficients.

A. (i) only

B. (i) and (ii) only

C. (i) and (iii) only

D. (ii) only

E. (ii) and (iii) only

5. (10 points) Consider the differential equation

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}.$$

Then the origin is

A. an attractor

B. a repeller

C. a saddle point

D. a spiral point

E. none of the above

6. Which of the following matrices are diagonalizable over the real numbers?

(i) $\begin{bmatrix} 2 & -5 \\ 3 & -6 \end{bmatrix}$ (ii) $\begin{bmatrix} 4 & 1 \\ 0 & 4 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & -1 & 3 \\ 0 & 5 & -2 \\ 0 & 0 & 7 \end{bmatrix}$ (iv) $\begin{bmatrix} 7 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 3 \end{bmatrix}$

A. (i) and (iii) only

B. (iii) and (iv) only

C. (i), (iii) and (iv) only

D. (i), (ii) and (iii) only

E. (i), (ii) and (iv) only

7. (10 points) A real 2×2 matrix A has an eigenvalue $\lambda_1 = 2 + i$ with corresponding eigenvector $\mathbf{v}_1 = \begin{bmatrix} 3 - i \\ 4 + i \end{bmatrix}$. Which of the following is the general **REAL** solution to the system of differential equations $\mathbf{x}'(t) = A\mathbf{x}(t)$?

A. $c_1 e^{2t} \begin{bmatrix} 3\cos t - \sin t \\ 4\cos t + \sin t \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 3\sin t + \cos t \\ 4\sin t - \cos t \end{bmatrix}$

B. $c_1 e^{2t} \begin{bmatrix} -3\cos t + \sin t \\ 4\cos t - \sin t \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 3\sin t - \cos t \\ 4\sin t - \cos t \end{bmatrix}$

C. $c_1 e^{2t} \begin{bmatrix} 3\cos t - \sin t \\ 4\cos t + \sin t \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 3\sin t - \cos t \\ 4\sin t - \cos t \end{bmatrix}$

D. $c_1 e^{2t} \begin{bmatrix} 3\cos t + \sin t \\ 4\cos t - \sin t \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 3\sin t + \cos t \\ 4\sin t - \cos t \end{bmatrix}$

E. $c_1 e^{2t} \begin{bmatrix} 3\cos t + \sin t \\ 4\cos t - \sin t \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 3\sin t - \cos t \\ 4\sin t + \cos t \end{bmatrix}$

8. Let $T : M_{2\times 2} \to M_{2\times 2}$ be a linear map defined as $A \mapsto A + A^T$.

(2 points) (1) Find $T\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\right)$

(4 points) (2) Find a basis for the range of T.

(4 points) (3) Find a basis for the kernel of T.

9. (6 points) (1) Find all the eigenvalues of matrix $A = \begin{bmatrix} 4 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 2 & 3 \end{bmatrix}$, and find a basis for the eigenspace corresponding to each of the eigenvalues.

(4 points) (2) Find an invertible matrix P and a diagonal matrix D such that

$$\begin{bmatrix} 4 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 2 & 3 \end{bmatrix} = PDP^{-1}.$$

10. (4 points) (1) Find the eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} -5 & 1 \\ 4 & -2 \end{bmatrix}.$$

(2 points) (2) Find a general solution to the system of differential equations

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} -5 & 1 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}.$$

(4 points) (3) Let $\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ be a particular solution to the initial value problem

$$\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} -5 & 1 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, \quad \begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}.$$

Find $x(1) + y(1)$.

Please write your answers of the 7 multiple choice questions in the following table.

Question	Answer
1. (10 points)	
2. (10 points)	
3. (10 points)	
4. (10 points)	
5. (10 points)	
6. (10 points)	
7. (10 points)	

Total Points: ____________________