MA 265, Spring 2023, Midterm I (GREEN)

INSTRUCTIONS:

1. **Write your answers of the seven multiple choice questions into the table on the last page. Show all your work on the questions and you may use the back of the test pages as scratch paper if needed.**
2. After you have finished the exam, hand in your test booklet to your instructor.

172	MWF	9:30AM	Ying Zhang	265	TR	1:30PM	Shiang Tang
173	MWF	10:30AM	Ying Zhang	276	TR	3:00PM	Yiran Wang
185	MWF	10:30AM	Seongjun Choi	277	TR	1:30PM	Yiran Wang
196	MWF	9:30AM	Seongjun Choi	281	MWF	2:30PM	Siamak Yassemi
201	MWF	2:30PM	Jing Wang	282	MWF	1:30PM	Siamak Yassemi
202	TR	4:30PM	Takumi Murayama	283	MWF	11:30AM	Ying Zhang
213	MWF	4:30PM	Eric Samperton	284	MWF	11:30AM	Seongjun Choi
214	MWF	3:30PM	Eric Samperton	285	MWF	7:30AM	Luming Zhao
225	MWF	11:30AM	Farrah Yhee	287	MWF	8:30AM	Luming Zhao
226	MWF	10:30AM	Farrah Yhee	288	MWF	12:30PM	Ping Xu
237	TR	10:30AM	Ying Liang	289	MWF	1:30PM	Ping Xu
238	TR	12:00PM	Ying Liang	290	MWF	11:30AM	Ping Xu
240	MWF	2:30PM	Ayan Maiti	291	MWF	12:30PM	Yevgeniya Tarasova
241	MWF	1:30PM	Ayan Maiti	292	MWF	11:30AM	Yevgeniya Tarasova
252	TR	12:00PM	Vaibhav Pandey	293	MWF	11:30AM	William Heinzer
253	TR	1:30PM	Vaibhav Pandey	294	TR	1:30PM	Guang Lin
264	TR	3:00PM	Shiang Tang	295	MWF	3:30PM	William Heinzer

3. NO CALCULATORS, BOOKS, NOTES, PHONES OR CAMERAS ARE ALLOWED on this exam. Turn off or put away all electronic devices.
4. When time is called, all students must put down their writing instruments immediately. You may remain in your seat while your instructor will collect the exam booklets.
5. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above instructions regarding academic dishonesty:

STUDENT NAME ______________________________

STUDENT SIGNATURE ______________________________

STUDENT PUID ______________________________

SECTION NUMBER ______________________________

1. (10 points) Suppose a 3×3 matrix $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ has determinant 2. What is the determinant of the matrix $\begin{bmatrix} a & b & 3c \\ g & h & 3i \\ d+2a & e+2b & 3f+6c \end{bmatrix}$?

A. 6

B. -6

C. 3

D. -3

E. 2

2. (10 points) Consider matrix $A = \begin{bmatrix} 1 & 2 & -1 & 5 & 0 \\ -2 & 0 & -2 & 2 & -4 \\ 3 & 4 & -1 & 9 & 2 \end{bmatrix}$, let a be the rank of A and b be the dimension of the null space of A, find $5a - 3b$.

A. 1

B. 9

C. 17

D. -7

E. None of the above

3. (10 points) Let $A = \begin{bmatrix} 1 & t & 2 \\ 3 & 3 & t-5 \\ 2 & 0 & 0 \end{bmatrix}$. Determine all the value of t such that the linear transform $T : \mathbb{R}^3 \to \mathbb{R}^3$ with $T(\mathbf{x}) = A\mathbf{x}$ is onto $\mathbb{R}^3$.

A. $t \neq 0, 5$

B. $t \neq 1, 6$

C. $t \neq 1, -6$

D. $t \neq -1, -6$

E. $t \neq -1, 6$

4. (10 points) Let A be an $m \times n$ matrix. Which of the following statements must be TRUE?

(i) If the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, then $A\mathbf{x} = \mathbf{b}$ has at most one solution.

(ii) If the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is one-to-one, then $m \geq n$.

(iii) If $m > n$, then the columns of A are linearly dependent.

(iv) If A has m pivot columns, then columns of A span $\mathbb{R}^m$.

A. (i) and (iii) only

B. (ii) and (iii) only

C. (ii) and (iv) only

D. (i), (ii) and (iv) only

E. (i) only

5. (10 points) Let A be a 5×5 matrix with column vectors $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4, \mathbf{a}_5$ satisfying $5\mathbf{a}_1 + \mathbf{a}_2 - 6\mathbf{a}_3 - 2\mathbf{a}_4 = 0$ and $2\mathbf{a}_2 - 7\mathbf{a}_3 + \mathbf{a}_4 - 3\mathbf{a}_5 = 0$. Which of the following statements must be TRUE?

A. $\det A \neq 0$

B. $\dim \operatorname{Nul} A = 2$

C. $\operatorname{rank} A \leq 3$

D. $\begin{bmatrix} 1 & 2 & -7 & 1 & -3 \end{bmatrix}^T$ is in $\operatorname{Nul} A$

E. The matrix equation $A\mathbf{x} = \mathbf{0}$ has a unique solution

6. (10 points) Find the adjugate of the matrix $\begin{bmatrix} -1 & 0 & -1 \\ -5 & 1 & -1 \\ 3 & 1 & -1 \end{bmatrix}$.

A. $\begin{bmatrix} 0 & -8 & -8 \\ -1 & 4 & 1 \\ 1 & 4 & -1 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 8 & 8 \\ 1 & -4 & -1 \\ -1 & -4 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 0 & -1 & 1 \\ -8 & 4 & 4 \\ -8 & 1 & -1 \end{bmatrix}$

D. $\begin{bmatrix} 0 & -1 & 1 \\ 8 & 4 & -4 \\ -8 & -1 & 1 \end{bmatrix}$

E. $\begin{bmatrix} 0 & 1 & 1 \\ 8 & 4 & -4 \\ -8 & -1 & -1 \end{bmatrix}$

7. (10 points) Which of the following collection of vectors spans $\mathbb{R}^4$?

A. $\left\{ \begin{bmatrix} 3 \\ 2 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 8 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 4 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 8 \\ 18 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 5 \\ 7 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \\ 8 \\ 10 \end{bmatrix}, \begin{bmatrix} 5 \\ 6 \\ 8 \\ 10 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 4 \\ 4 \end{bmatrix} \right\}$

8. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation for which

$$T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}3\\4\end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix}-1\\1\end{bmatrix}\right) = \begin{bmatrix}-1\\-2\end{bmatrix}.$$

(4 points)(1) Find $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)$ and $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$.

(4 points) (2) Let A be the standard matrix of T, find A and A^{-1}.

(2 points)(3) Let $\mathbf{b} = \begin{bmatrix}7\\9\end{bmatrix}$, find all the vectors $\mathbf{x}$ in $\mathbb{R}^2$ such that $T(\mathbf{x}) = \mathbf{b}$.

9. Consider the linear system

$$\begin{aligned} x \qquad\qquad\quad - z &= \quad 1 \\ x + y \ \ + (h-1)z &= \quad 3 \\ 2y + (h^2-3)z &= h+1 \end{aligned}$$

(4 points)(1) Find a row echelon form for the augmented matrix of the system.

(2 points)(2) For which value(s) of h does this system have an infinite number of solutions?

(2 points)(3) For which value(s) of h does this system have no solution?

(2 points)(4) For which value(s) of h does this system have a unique solution?

10. Let

$$A = \begin{bmatrix} 1 & 0 & 2 & 4 & 11 \\ 1 & 0 & 5 & 13 & 20 \\ 2 & 0 & 4 & 12 & 22 \\ 3 & 0 & 2 & 0 & 21 \end{bmatrix}.$$

(4 points)(1) Find the REDUCED row echelon form for the matrix A.

(3 points)(2) Find a basis for the column space of A.

(3 points)(3) Find a basis for the null space of A.

Please write your answers of the 7 multiple choice questions in the following table.

Question	Answer
1. (10 points)	
2. (10 points)	
3. (10 points)	
4. (10 points)	
5. (10 points)	
6. (10 points)	
7. (10 points)	

Total Points: ________________________