MA 265, Fall 2022, Midterm I (GREEN)

INSTRUCTIONS:

1. **Write your answers of the seven multiple choice questions into the table on the last page. Show all your work on the questions and you may use the back of the test pages as scratch paper if needed.**
2. After you have finished the exam, hand in your test booklet to your instructor.

101	MWF	10:30AM	Ying Zhang	600	MWF	1:30PM	Seongjun Choi
102	MWF	9:30AM	Ying Zhang	601	MWF	1:30PM	Ayan Maiti
153	MWF	11:30AM	Ying Zhang	650	MWF	10:30AM	Yevgeniya Tarasova
154	MWF	11:30AM	Daniel Tuan-Dan Le	651	MWF	9:30AM	Yevgeniya Tarasova
205	TR	1:30PM	Oleksandr Tsymbaliuk	701	MWF	3:30PM	Seongjun Choi
206	TR	3:00PM	Oleksandr Tsymbaliuk	702	MWF	11:30AM	Yiran Wang
357	MWF	1:30PM	Yiran Wang	703	MWF	12:30PM	Ke Wu
410	TR	1:30PM	Arun Debray	704	MWF	1:30PM	Ke Wu
451	TR	10:30AM	Arun Debray	705	MWF	12:30PM	Seongjun Choi
501	TR	12:00PM	Vaibhav Pandey	706	TR	1:30PM	Vaibhav Pandey
502	MWF	10:30AM	Ayan Maiti	707	MWF	3:30PM	Siamak Yassemi
				708	MWF	2:30PM	Siamak Yassemi

3. NO CALCULATORS, BOOKS, NOTES, PHONES OR CAMERAS ARE ALLOWED on this exam. Turn off or put away all electronic devices.
4. When time is called, all students must put down their writing instruments immediately. You may remain in your seat while your instructor will collect the exam booklets.
5. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above instructions regarding academic dishonesty:

STUDENT NAME ______________________

STUDENT SIGNATURE ______________________

STUDENT PUID ______________________

SECTION NUMBER ______________________

1. (10 points) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix}$, and $C = AB^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $a+b+c+d =$

A. -7

B. 8

C. 7

D. -8

E. 0

2. (10 points) Let L be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ whose standard matrix is $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 2 & 3 & k \end{bmatrix}$ where k is a real number. Find all values of k such that L is one-to-one.

A. $k \neq 1$

B. $k \neq 2$

C. $k \neq 3$

D. $k \neq 4$

E. $k \neq 5$

3. (10 points) Which of the following statements is/are always TRUE?

(i) If A is a singular 8×8 matrix, then its last column must be a linear combination of the first seven columns.

(ii) Let A be a 5×7 matrix such that $A \cdot \mathbf{x} = \mathbf{b}$ is consistent for any $\mathbf{b} \in \mathbb{R}^5$, and let B be a 7×11 matrix such that $B \cdot \mathbf{x} = \mathbf{c}$ is consistent for any $\mathbf{c} \in \mathbb{R}^7$. Then, the matrix equation $AB \cdot \mathbf{x} = \mathbf{b}$ is consistent for any $\mathbf{b} \in \mathbb{R}^5$.

(iii) For any $m \times n$ matrix A, the dimension of the null space of A equals the dimension of the null space of its transpose A^T.

(iv) If A is an $m \times n$ matrix, then the set $\{A \cdot \mathbf{x} | \mathbf{x} \in \mathbb{R}^n\}$ is a subspace of $\mathbb{R}^m$.

A. (i) only

B. (i) and (ii) only

C. (iv) only

D. (ii) and (iv) only

E. (iii) and (iv) only

4. (10 points) Compute the determinant of the given matrix $\begin{bmatrix} 5 & -7 & 2 & 2 \\ 0 & 3 & 0 & -4 \\ -5 & -8 & 0 & 3 \\ 0 & 5 & 0 & -6 \end{bmatrix}$.

A. -20

B. 20

C. 18

D. 2

E. 0

5. (10 points) Which of the following statements is always TRUE?

A. If A is an $n \times n$ matrix with all entries being positive, then $det(A) > 0$.

B. If A and B are two $n \times n$ matrices with $det(A) > 0$ and $det(B) > 0$, then also $det(A + B) > 0$.

C. If A and B are two $n \times n$ matrices such that $AB = 0$, then both A and B are singular.

D. If rows of an $n \times n$ matrix A are linearly independent, then $det(A^T A) > 0$.

E. If A is an $n \times n$ matrix with $A^2 = I_n$, then $det(A) = 1$.

6. (10 points) Let $A = \begin{bmatrix} 1 & 2 & 6 \\ 2 & 6 & 3 \\ 3 & 8 & 10 \end{bmatrix}$ and let its inverse $A^{-1} = [b_{ij}]$. Find b_{12}.

A. 14

B. -14

C. 1

D. -1

E. 6

7. (10 points) Let $\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$, $\boldsymbol{v}_2 = \begin{bmatrix} -2 \\ -3 \\ 1 \end{bmatrix}$ and $\boldsymbol{x} = \begin{bmatrix} -4 \\ -5 \\ 13 \end{bmatrix}$, and $\boldsymbol{B} = \{\boldsymbol{v}_1, \boldsymbol{v}_2\}$. Then $\boldsymbol{B}$ is a basis for $H = \text{span}\{\boldsymbol{v}_1, \boldsymbol{v}_2\}$. Determine if $\boldsymbol{x}$ is in H, and if it is, find the coordinate vector of $\boldsymbol{x}$ relative to $\boldsymbol{B}$.

A. $[\boldsymbol{x}]_{\boldsymbol{B}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

B. $[\boldsymbol{x}]_{\boldsymbol{B}} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

C. $[\boldsymbol{x}]_{\boldsymbol{B}} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

D. $[\boldsymbol{x}]_{\boldsymbol{B}} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

E. $[\boldsymbol{x}]_{\boldsymbol{B}} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

8. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation for which

$$T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}3\\2\\1\end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix}1\\2\end{bmatrix}\right) = \begin{bmatrix}1\\0\\2\end{bmatrix}.$$

(4 points)(1) Let A be the standard matrix of T, find A.

(2 points)(2) Find the image of the vector $\mathbf{u} = \begin{bmatrix}1\\3\end{bmatrix}$.

(4 points)(3) Is the vector $\mathbf{b} = \begin{bmatrix}0\\-2\\5\end{bmatrix}$ in the range of T? If so, find all the vectors $\mathbf{x}$ in $\mathbb{R}^2$ such that $T(\mathbf{x}) = \mathbf{b}$.

9. Consider the linear system

$$\begin{aligned} x + 2y \; + 3z &= \quad 2 \\ y \; + az &= \; -4 \\ 2x + 5y \; + a^2 z &= \; a - 3 \end{aligned}$$

(4 points)(1) Find a row echelon form for the augmented matrix of the system.

(2 points)(2) For which value(s) of a does this system have an infinite number of solutions?

(2 points)(3) For which value(s) of a does this system have no solution?

(2 points)(4) For which value(s) of a does this system have a unique solution?

10. Let

$$A = \begin{bmatrix} 1 & 2 & 0 & -1 & 2 \\ 2 & 3 & 1 & -3 & 7 \\ 3 & 4 & 1 & -3 & 9 \end{bmatrix}.$$

(5 points)(1) Find the REDUCED row echelon form for the matrix A.

(5 points)(2) Find a basis for the null space of A.

Please write your answers of the 7 multiple choice questions in the following table.

Question	Answer
1. (10 points)	
2. (10 points)	
3. (10 points)	
4. (10 points)	
5. (10 points)	
6. (10 points)	
7. (10 points)	

Total Points: ____________________