May 3, 2023

# MA262 - FINAL EXAM

Green Exam: TEST NUMBER $\boxed{\mathbf{01}}$

## Instructions

(1) **DO NOT OPEN THIS EXAM BOOKLET UNTIL TOLD TO DO SO**.

(2) Before you open this exam booklet, fill in the information below and use a #2 pencil to fill in the required information on your scantron.

(3) On your scantron, write your 10-digit PUID (starting with "00" from left to right) and your 4-digit Recitation section number (starting with "0" from left to right).

(4) This Green exam is TEST NUMBER $\boxed{\mathbf{01}}$.

(5) Once you are allowed to open this booklet, check to make sure you have a complete exam. There are **15** different exam pages, including this cover page.

(6) Do any necessary work for each problem in the space provided or on the back of the pages of this exam booklet. No extra paper is allowed. **Circle** your answers in this exam booklet in case of a lost scantron.

(7) There are **20** problems, each worth 10 points. The maximum possible score is 200. No partial credit will be given.

(8) After you finish your exam, hand in both your scantron and your exam booklet to your instructor, your TA, or one of the proctors.

## Academic Honesty

- Do not seek or obtain any assistance from anyone to answer questions on this exam.
- Do not talk during the exam. Any questions should be directed to your instructor or TA.
- Do not consult notes, books, or calculators during the exam. Do not handle phones, cameras, or any other electronic devices until after you have finished your exam, handed it in to your instructor, your TA or proctor, and left the room.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty may include an **F** in this course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

*I have read and understand the above statements regarding academic honesty* :

**Student Name** ____________________ **PUID** ____________________

**Student Signature** ________________________________________

**Recitation Section #** __________ **TA Name** ____________________

**1.** Let $y$ be the solution of the initial value problem

$$x^3 y' + 3x^2 y = e^x, \quad y(1) = 2e\,.$$

What is $y(2)$?

**A.** $\frac{1}{8}\left(e^2 - 2e\right)$

**B.** $\frac{1}{8}\left(e^2 + e\right)$

**C.** $\frac{e}{8}$

**D.** $\frac{1}{8}\left(e^2 - 2\right)$

**E.** $\frac{e^2}{8} + e$

**2.** Find the general solution to the differential equation

$$(e^x \sin y - 2y \sin x) + (e^x \cos y + 2\cos x + 2y)\,\frac{dy}{dx} = 0\,.$$

**A.** $e^x \sin y + 2y \cos x + y^2 = C$

**B.** $e^x \cos y + 2y \sin x + 2y = C$

**C.** $e^x \sin y - 2y \cos x + y^2 = C$

**D.** $e^x \cos y - 2y \sin x + 2y = C$

**E.** $e^x \sin x + 2x \cos y - y^2 = C$

**3.** If $\dfrac{du}{dt} = 2tu$ and $u(0) = 4$, what is $u(2)$?

**A.** $e^4 + 3$

**B.** $4e^4$

**C.** $2e^4$

**D.** $4e^2$

**E.** $e^2 + 3$

**4.** Find the general solution of this Bernoulli differential equation: $\dfrac{dy}{dx} - \dfrac{1}{x}\, y = 3xy^2$ .

**A.** $y(x) = \dfrac{x}{C - x^3}$

**B.** $y(x) = \dfrac{1}{x}\left(C - x^2\right)$

**C.** $y(x) = \dfrac{x^2}{C - x^3}$

**D.** $y(x) = \dfrac{C - x^3}{x^2}$

**E.** $y(x) = \dfrac{x}{C - x}$

**5.** Consider the autonomous differential equation

$$\frac{dx}{dt} = \left(x^2 - 9\right)(x+1)\,.$$

Which one of the following statements is **TRUE**?

**A.** $x = 3$ and $x = -3$ are **stable** critical points.

**B.** $x = 3$ and $x = -3$ are **semistable** critical points.

**C.** One of $x = -1$, $x = 3$ is a **semistable** critical point.

**D.** $x = -1$ is an **unstable** critical point.

**E.** $x = -3$ is an **unstable** critical point and $x = -1$ is a **stable** critical point.

**6.** Find all the values of $k$ such that the matrix $\begin{bmatrix} 1 & 3 & 1 \\ 2 & k & 0 \\ 3 & 0 & k \end{bmatrix}$ is invertible.

**A.** $k \neq 0$ and $k \neq 3$

**B.** $k \neq 0$ and $k \neq 9$

**C.** $k \neq 1$ and $k \neq 0$

**D.** $k \neq -3$ and $k \neq 3$

**E.** It is invertible for all values of $k$

**7.** Suppose the linear system $A\mathbf{x} = \mathbf{b}$ is **consistent** whenever

$$A = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 2 & -3 \\ 3 & 5 & -4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -1 \\ \alpha \\ \beta \end{bmatrix}.$$

Which one of the following statements must always be TRUE?

**A.** The system has a unique solution

**B.** $\beta = \alpha$

**C.** $\beta = 1 + 2\alpha$

**D.** $\beta = 1 - 2\alpha$

**E.** $A$ is invertible

**8.** Consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 2 & 1 & 0 & 0 \\ 0 & 1 & -3 & 1 \end{bmatrix}.$$

Which one of the following statements is TRUE?

**A.** The columns of $A$ are linearly independent.

**B.** The rows of $A$ are linearly independent.

**C.** The rank of $A$ is 2.

**D.** The null space of $A$ has dimension 2.

**E.** The first two rows of $A$ form a basis for the Row Space of $A$.

**9.** Find the dimension of the solution space of the system of equations $A\mathbf{x} = \mathbf{0}$ when

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 2 & 0 & 3 \\ 0 & -2 & 2 & 8 & 4 \end{bmatrix}.$$

**A.** 0

**B.** 2

**C.** 1

**D.** 3

**E.** 4

**10.** Which of these subsets of $\mathbb{R}^2$ form vector spaces with respect to the usual operations of addition and scalar multiplication?

(I) $\Big\{(x, y) : \quad xy = 0\Big\}$

(II) $\Big\{(x, y) : \quad x + y > 0\Big\}$

(III) $\Big\{(x, y) : \quad x + y = 0\Big\}$

(IV) $\Big\{(x, y) : \quad x + y = 1\Big\}$

(V) $\left\{(x, y) : \quad \begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\}$

**A.** (I) and (II) only

**B.** (I) and (III) only

**C.** (III) and (V) only

**D.** (IV) and (V) only

**E.** (I), (II) and (IV) only

**11.** Which of the following set of vectors below is a basis for the Column Space of

$$\begin{bmatrix} 1 & -2 & 0 & 1 & 0 \\ 2 & -4 & 0 & 2 & 0 \\ -1 & 2 & 0 & 3 & 4 \end{bmatrix} ?$$

**A.** $\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} \right\}$

**B.** $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \right\}$

**C.** $\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$

**D.** $\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} \right\}$

**E.** $\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix} \right\}$

**12.** Find the general solution of the homogeneous differential equation

$$(D-1)^2\,(D^2-2D+2)\,y = 0\,.$$

Note: The Characteristic equation is $(r-1)^2(r^2-2r+2)=0$.

**A.** $y = C_1 e^x + C_2 x e^x + C_3 \cos x + C_4 \sin x$

**B.** $y = C_1 e^x + C_2 e^x \cos x + C_3 e^x \sin x$

**C.** $y = C_1 e^x + C_2 e^x + C_3 \cos x + C_4 \sin x$

**D.** $y = C_1 e^x + C_2 x e^x + C_3 e^x \cos 2x + C_4 e^x \sin 2x$

**E.** $y = C_1 e^x + C_2 x e^x + C_3 e^x \cos x + C_4 e^x \sin x$

**13.** Solve this initial value problem: $\begin{cases} y''' + 3y'' = 0 \\ y(0) = 2,\ y'(0) = 1,\ y''(0) = 9 \end{cases}$.

**A.** $y = 1 - 2x + e^{3x}$

**B.** $y = 2 - 2x - e^{-3x}$

**C.** $y = -5x - \frac{9}{2}x^2 + 2e^{3x}$

**D.** $y = 1 + 4x + e^{-3x}$

**E.** $y = 2 - e^{-3x}$

**14.** Using the method of Undetermined Coefficients, which of the following is the correct form of a particular solution $y_p$ to the nonhomogeneous differential equation

$$y^{(4)} - y = xe^x + 3\cos x - 4x \ ?$$

**A.** $y_p = x(A + Bx)e^x + (C\cos x + D\sin x) + Ex$

**B.** $y_p = (A + Bx)e^x + (C\cos x + D\sin x) + E + Fx$

**C.** $y_p = (A + Bx)e^x + x(C\cos x + D\sin x) + x(E + Fx)$

**D.** $y_p = x(A + Bx)e^x + x(C\cos x) - Dx$

**E.** $y_p = x(A + Bx)e^x + x(C\cos x + D\sin x) + E + Fx$

**15.** Solve this initial value problem $\begin{cases} y'' - 4y = 8x \\ \\ y(0) = 0, \ y'(0) = 0 \end{cases}$.

**A.** $y = 0$

**B.** $y = e^{2x} - e^{-2x} + 8x$

**C.** $y = \frac{1}{2}e^{2x} - \frac{1}{2}e^{-2x} - 2x$

**D.** $y = 2e^{4x} + 6xe^{4x} - 2x$

**E.** $y = e^{4x} - 4x$

**16.** The differential equation $y'' + (\tan x)\, y' = 0$ has two linearly independent solutions $y_1(x) = \sin x$ and $y_2(x) = 1$. If we use the method of Variation of Parameters to find a particular solution $y_p(x) = u_1(x)y_1(x) + u_2(x)y_2(x)$ for the corresponding nonhomogeneous equation

$$y'' + (\tan x)\, y' = \cos^2 x\,,$$

then which of the following is a possible function $u_1(x)$?

**A.** $u_1(x) = \sin x$

**B.** $u_1(x) = -\dfrac{1}{2}\sin^2 x$

**C.** $u_1(x) = \cos^2 x$

**D.** $u_1(x) = \dfrac{1}{2}\cos^2 x$

**E.** $u_1(x) = \cos x$

**17.** Given that the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ has eigenvalues $\lambda_1 = -1$ and $\lambda_2 = 3$, the general solution to $\mathbf{x}' = A\mathbf{x}$ is

**A.** $\mathbf{x}(t) = C_1 e^{-t} \begin{bmatrix} -1 \\ 1 \end{bmatrix} + C_2 e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

**B.** $\mathbf{x}(t) = C_1 e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 e^{3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

**C.** $\mathbf{x}(t) = C_1 e^{t} \begin{bmatrix} -1 \\ 1 \end{bmatrix} + C_2 e^{-3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

**D.** $\mathbf{x}(t) = C_1 e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

**E.** $\mathbf{x}(t) = C_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 e^{3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

**18.** Let $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ be the solution to the initial value problem

$$\mathbf{x}'(t) = \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix} \mathbf{x}(t), \quad \mathbf{x}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Then $x_2(1) =$

**A.** $2e + e^4$

**B.** $-e + e^4$

**C.** $e$

**D.** $e^4$

**E.** $2e - 2e^4$

**19.** Which of the following statement(s) is/are TRUE?

**(I)** The vector $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is an eigenvector for $A = \begin{bmatrix} 1 & 3 \\ -3 & -5 \end{bmatrix}$.

**(II)** The origin is a *saddle point* for the linear system $\mathbf{x}' = \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} \mathbf{x}$.

**(III)** Given that the general solution to the linear system $\mathbf{x}' = \begin{bmatrix} 5 & 2 \\ 1 & 4 \end{bmatrix} \mathbf{x}$ is

$$\mathbf{x}(t) = C_1 e^{3t} \begin{bmatrix} -1 \\ 1 \end{bmatrix} + C_2 e^{6t} \begin{bmatrix} 2 \\ 1 \end{bmatrix},$$

the origin is a *nodal source.*

**A.** Only **(II)**

**B.** Only **(I)** and **(III)**

**C.** Only **(III)**

**D.** Only **(II)** and **(III)**

**E.** All are TRUE

**20.** Find the solution to the initial value problem

$$\mathbf{x}' = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 5 \\ 1 \end{bmatrix}.$$

**A.** $\mathbf{x}(t) = e^{2t} \begin{bmatrix} 5 - 3t \\ 1 - 3t \end{bmatrix}$

**B.** $\mathbf{x}(t) = e^{2t} \begin{bmatrix} 5 + 3t \\ 1 + 3t \end{bmatrix}$

**C.** $\mathbf{x}(t) = e^{2t} \begin{bmatrix} 5 + 2t \\ 1 + 2t \end{bmatrix}$

**D.** $\mathbf{x}(t) = e^{2t} \begin{bmatrix} 5 + t \\ 1 + t \end{bmatrix}$

**E.** $\mathbf{x}(t) = e^{2t} \begin{bmatrix} 5 + 4t \\ 1 + 4t \end{bmatrix}$