MA 262
FINAL EXAM INSTRUCTIONS
May 2, 2002

NAME ______________________ INSTRUCTOR ______________________

1. You must use a #2 pencil on the mark–sense sheet (answer sheet).

2. Make sure the cover of your question booklet is the same color as your answer sheet.

3. On the mark-sense sheet, fill in the instructor's name and the course number.

4. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces.

5. Fill in the SECTION NUMBER boxes with the division and section number of your class. For example, for division 02, section 03, fill in 0203 and blacken the corresponding circles, including the circles for the zeros. (If you do not know your division and section number ask your instructor.)

6. Sign the mark–sense sheet.

7. Fill in your name and your instructor's name on the question sheets above.

8. There are 25 questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25. Do all your work on the question sheets. Turn in both the mark–sense sheets and the question sheets when you are finished.

9. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

10. NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

1. Solutions to $(2xy+\cos y)+(x^2-x\sin y-2)\frac{dy}{dx}=0$ satisfy

A. $x^2y+x\cos y-2y=c$

B. $x^2y+\cos y-2y=c$

C. $x^2y+x\sin y-2=c$

D. $x^2y-x\cos y-2x=c$

E. $x^2y^2+x\cos y-2x=c$

2. The general solution to $xy'+y=e^{5x}$ is

A. $y=\frac{1}{5}e^{5x}+c$

B. $y=\frac{1}{6}e^{5x}+ce^{-x}$

C. $y=\frac{1}{5x}e^{5x}+cx^{-1}$

D. $y=ce^{5x}$

E. $y=\frac{5}{x}e^{5x}+c$

3. The substitution $v=y/x$ transforms the equation $\frac{dy}{dx}=\sin(y/x)$ into

A. $v'=\sin(v)$

B. $v'=x\sin(v)$

C. $v'+v=\sin(v)$

D. $xv'+v=\sin(v)$

E. $v'+xv=\sin(v)$

4. If $x = x(t)$ is the solution to the initial value problem

$$\frac{d^2x}{dx^2} + x = 2te^t, \qquad x(0) = -1, \ x'(0) = 0,$$

then $x(1) =$

A. e

B. $\frac{1}{e}$

C. -1

D. 1

E. 0

5. A fish tank contains 20 gallons of a salt solution with a concentration of 5 grams of salt per gallon. A salt solution with a concentration of 10 grams/gallon is added to the tank at a rate of 2 gallons per minute. At the same time, water is drained from the tank at a rate of 2 gallons per minute. How many grams of salt are in the tank after 10 minutes?

A. $200e^{-1} + 100$

B. $200 - 100e^{-1}$

C. $200e - 100$

D. 100

E. 200

6. The subspace of $\mathbb{R}^3$ spanned by the vectors $(1,1,1)$, $(1,1,0)$, $(0,0,1)$, $(3,2,1)$ has dimension

A. 0

B. 1

C. 2

D. 3

E. 4

7. Find a basis for the null space of the matrix $\begin{bmatrix} 1 & 0 & 2 & -1 \\ 2 & -1 & 5 & -2 \\ 0 & 1 & -1 & 0 \end{bmatrix}$.

A. $\left\{ \begin{pmatrix} 1 \\ -1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \\ 1 \\ 0 \end{pmatrix} \right\}$

B. $\left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix} \right\}$

C. $\left\{ \begin{pmatrix} -2 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

D. $\left\{ \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \right\}$

E. $\left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix} \right\}$

8. If $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation satisfying

$$T(1,0,0) = (0,0,-1), \quad T(0,1,0) = (1,1,0), \quad T(0,0,1) = (1,1,2),$$

then $T(1,2,3) =$

A. $(5,5,5)$

B. $(2,1,0)$

C. $(2,-1,1)$

D. $(0,0,0)$

E. $(1,2,3)$

9. For which value(s) of k are the vectors $(2, -k, 0)$, $(1, 2, 2)$, and $(0, 1, -k)$ linearly *dependent*?

A. No values of k.

B. $k = -2$

C. $k \neq -2$

D. $k = 0$

E. All values of k.

10. For which value(s) of k are the vectors $(2, -k, 0)$ and $(1, 2, 2)$ linearly *dependent*?

A. No values of k.

B. $k = -2$

C. $k \neq -2$

D. $k = 0$

E. All values of k.

11. For which value(s) of k are the vectors $(2, -k, 0)$, $(1, 2, 2)$, $(0, 1, -k)$, and $(0, 1, k)$ linearly *dependent*?

A. No values of k.

B. $k = -2$

C. $k \neq -2$

D. $k = 0$

E. All values of k.

12. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{bmatrix}$, and $B = A^{-1}$, then the entry b_{23} of B is

A. 0

B. -4

C. 4

D. $1/6$

E. $-2/3$

13. Determine all values of k so that the following system has **no solution**.

$$\begin{aligned} x_1 + x_2 + 3x_3 &= 1 \\ 2x_1 + 4x_2 + 5x_3 &= 1 \\ x_1 - x_2 + k^2 x_3 &= -k \end{aligned}$$

A. $k \neq \pm 2$

B. $k = \pm 2$

C. $k = 2$

D. $k = -2$

E. $-2 < k < 2$

14. If one solution of $y'' + y' - 2y = f(x)$ is $y(x) = \ln x$, then the general solution is

A. $c_1 \ln x$

B. $c_1 \ln x + c_2 e^{x} + c_3 e^{-2x}$

C. $c_1 e^{x} + c_2 e^{-2x} + \ln x$

D. $c_1 e^{x} + c_2 e^{-2x}$

E. $c_1 e^{-x} + c_2 e^{2x} + \ln x$

15. If $y(x)$ is the solution of $y'' - y' - 2y = 0$ satisfying $y(0) = 1$ and $y'(0) = -1$, then $y(1) =$

A. $e^{-1} + 2e^2$

B. e^{-1}

C. e^2

D. $e^2 - e^{-1}$

E. $2e^2$

16. Find all values of the exponent r such that $y = x^r$ is a solution to the equation $x^2y'' + 5xy' + 4y = 0$.

A. $r = -2$

B. $r = -1, -4$

C. $r = 1, 4$

D. $r = \pm 4$

E. $r = -\frac{5}{2} \pm \frac{3}{2}i$

17. The general solution of $y^{(iv)} + 2y'' + y = 0$ is

A. $y = c_1 \cos t + c_2 \sin t$

B. $y = c_1 t \cos t + c_2 t \sin t$

C. $y = c_1 e^t + c_2 te^t + c_3 \cos t + c_4 \sin t$

D. $y = c_1 e^t + c_2 te^t + c_3 e^{-t} + c_4 te^{-t}$

E. $y = c_1 \cos t + c_2 \sin t + c_3 t \cos t + c_4 t \sin t$

18. In the method of undetermined coefficients, the appropriate form of the particular solution for the equation $y''' - 4y' = 3\cos t + e^{-2t}$ is

A. $y_p(t) = A\cos t + B\sin t + Ce^{-2t}$

B. $y_p(t) = A\cos t + Be^{-2t}$

C. $y_p(t) = A\cos t + Bte^{-2t}$

D. $y_p(t) = A\cos t + B\sin t + Cte^{-2t}$

E. $y_p(t) = At\cos t + Bt\sin t + Ce^{-2t}$

19. A particular solution for $y'' + 2y' + y = x^{-1}e^{-x}$ is

A. $(\ln x - x^2)e^{-x}$

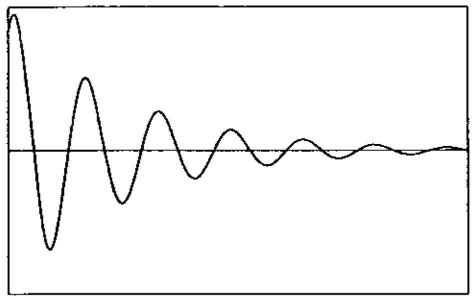
B. $Ax^{-1}e^{-x}$

C. Axe^{-x}

D. $e^{-x}\ln x$

E. $xe^{-x}\ln x$

20. For which values of the parameter α will the equation $y'' + \alpha y' + y = 0$ have all its solutions tending to zero as t tends to infinity in a manner similar to that depicted in the following graph?



A. $\alpha < 2$

B. $\alpha = 0$

C. $0 < \alpha < 2$

D. $\alpha > 2$

E. all α

21. The eigenvalues of the matrix $\begin{bmatrix} 2 & 0 & 0 \\ 1 & 3 & 4 \\ 0 & -1 & -1 \end{bmatrix}$ are

A. -2,1,-1

B. 3,3,-1

C. 2,2,1

D. 2,1,1

E. 1,-1,2

22. The general solution of the linear system of differential equations

$$\begin{aligned} x_1' &= x_1 + 2x_2 \\ x_2' &= 4x_1 + 3x_2 \end{aligned}$$

is equal to

A. $c_1 \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix} + c_2 \begin{bmatrix} e^{5t} \\ 2e^{5t} \end{bmatrix}$

B. $c_1 \begin{bmatrix} e^{-t} \\ e^{-t} \end{bmatrix} + c_2 \begin{bmatrix} e^{5t} \\ -2e^{5t} \end{bmatrix}$

C. $c_1 \begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix} + c_2 \begin{bmatrix} e^{5t} \\ -2e^{5t} \end{bmatrix}$

D. $c_1 \begin{bmatrix} e^{-t} \\ e^{-t} \end{bmatrix} + c_2 \begin{bmatrix} e^{5t} \\ 2e^{5t} \end{bmatrix}$

E. None of the above.

23. The function $x_2(t)$ determined by the initial value problem

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= -x_1 \end{aligned}$$

with initial conditions $x_1(0) = 1$ and $x_2(0) = 1$ is given by

A. $x_2 = -\sin t + \cos t$

B. $x_2 = \sin t + \cos t$

C. $x_2 = \frac{1}{2}(e^t + e^{-t})$

D. $x_2 = \cos t$

E. $x_2 = ie^{it} - ie^{-it}$

24. The 2×2 matrix $A = \begin{bmatrix} -3 & -1 \\ 2 & -1 \end{bmatrix}$ has complex eigenvalues $r = -2 \pm i$. An eigenvector corresponding to $r = -2 + i$ is $\begin{pmatrix} 1 \\ -1 - i \end{pmatrix}$. The system

$$\vec{x}' = A\vec{x} + \begin{pmatrix} 3 \\ -4 \end{pmatrix} e^{-2t}$$

has one solution given by $\vec{x}(t) = \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$. What is the general solution to the system?

A. $c_1 \begin{pmatrix} 1 \\ -1 - i \end{pmatrix} e^{(-2+i)t} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{2t}$

B. $c_1 \begin{pmatrix} \cos t \\ \sin t - \cos t \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} \sin t \\ -\sin t - \cos t \end{pmatrix} e^{-2t} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$

C. $c_1 \begin{pmatrix} \cos t \\ -\cos t \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} 0 \\ -\sin t \end{pmatrix} e^{-2t} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$

D. $c_1 \begin{pmatrix} 1 \\ -1 - i \end{pmatrix} e^{(-2+i)t} + c_2 \begin{pmatrix} -1 \\ 1 + i \end{pmatrix} e^{(-2-i)t} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$

E. $c_1 \begin{pmatrix} \cos t \\ -\cos t \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} 0 \\ -\sin t \end{pmatrix} e^{-2t} + c_3 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$

25. If we know that the determinant of a 3×3 matrix A is zero, then we can deduce that

A. Any system of the form $A\vec{x} = \vec{b}$ has infinitely many solutions.

B. The system $A\vec{x} = 0$ has only the zero solution.

C. The system $A\vec{x} = 0$ has a non-zero solution.

D. $\lambda = 1$ is an eigenvalue for A.

E. A^{-1} exists.