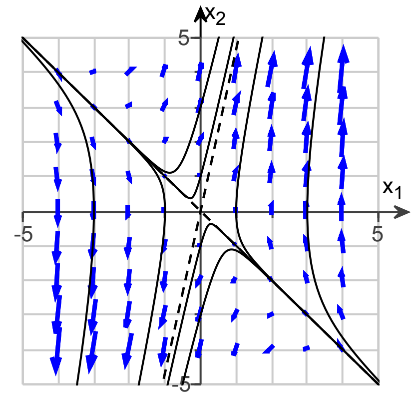
Student: ____________________ **Date:** ____________________	**Instructor:** Antonio PurdueMath **Course:** MA262 Fall2020 Coordinator course	**Assignment:** Online Final Exam

1. The phase portrait to the right corresponds to a linear system of the form $\mathbf{x}' = \mathbf{Ax}$ in which the matrix **A** has two linearly independent eigenvectors. Determine the nature of the eigenvalues of the system.



Click here to view page 1 of Gallery of Typical Phase Portraits for the System x'=Ax: Nodes[7]
Click here to view page 2 of Gallery of Typical Phase Portraits for the System x'=Ax: Nodes[8]
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The phase portrait to the right corresponds to a linear system of the form in which the matrix **A** has two linearly independent eigenvectors. Determine the nature of the eigenvalues of the system.

Click here to view page 1 of Gallery of Typical Phase Portraits for the System x'=Ax: Nodes
Click here to view page 2 of Gallery of Typical Phase Portraits for the System x'=Ax: Nodes
Click here to view page 3 of Gallery of Typical Phase Portraits for the System x'=Ax: Nodes

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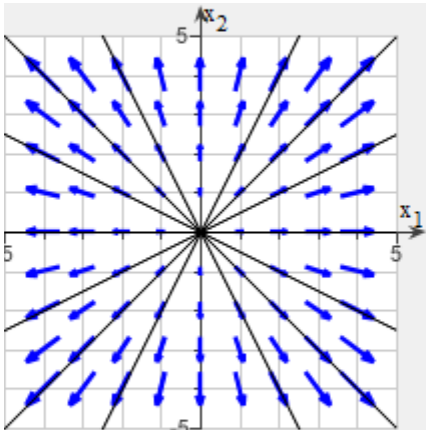
The system shows (1) ____________ and its eigenvalues are (2) ____________ and it has two linearly independent eigenvectors (3) ____________

	distinct, real, with one zero,
a center	distinct, negative, and real,
a saddle point	complex with a positive real part,
a spiral sink	complex with a negative real part,
a proper nodal source	distinct, opposite in sign, and real,
parallel lines	repeated, real, and zero,
a proper nodal sink	distinct, positive, and real,
an improper nodal sink	purely imaginary,
a spiral source	repeated, negative, and real,
an improper nodal source	repeated, positive, and real,

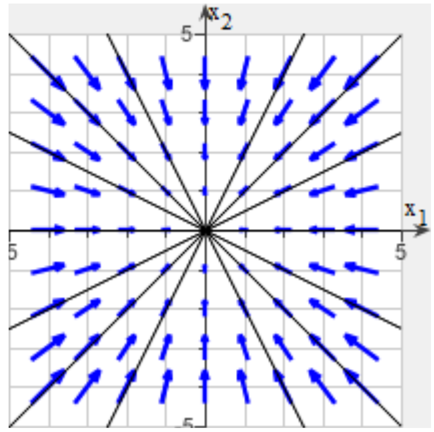
The system shows and its eigenvalues are and it has two linearly independent eigenvectors
that are approximately @MATX{{1};{-1}} and @MATX{{1};{2}}.
about which nothing else can be inferred.
that are approximately @MATX{{1};{0}} and @MATX{{1};{2}}.
that are approximately @MATX{{1};{0}} and @MATX{{0};{1}}.

1: Test

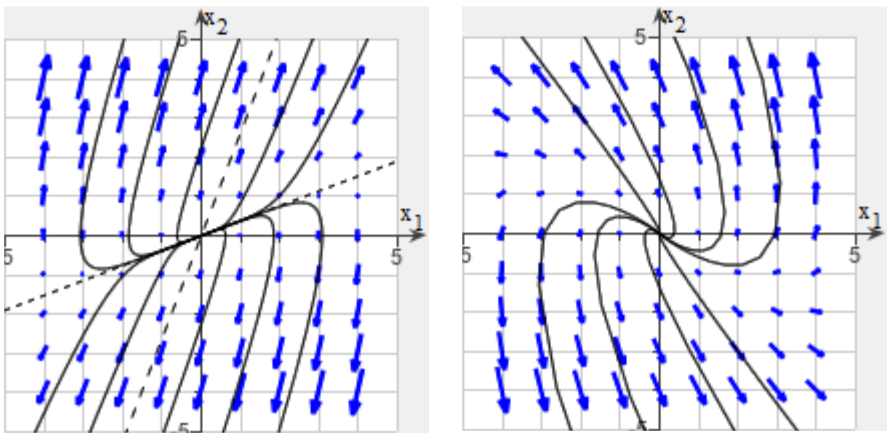
Gallery of Typical Phase Portraits for the System x'=Ax: Nodes



Proper Nodal Source: A repeated positive real eigenvalue with two linearly independent eigenvectors.



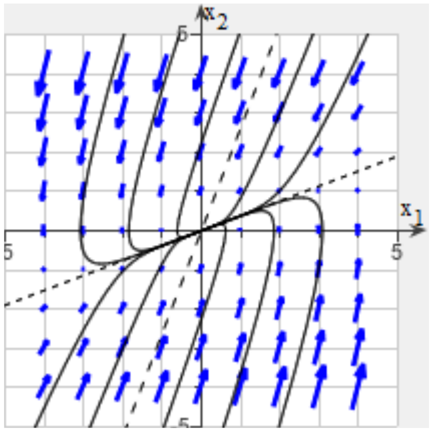
Proper Nodal Sink: A repeated negative real eigenvalue with two linearly independent eigenvectors.

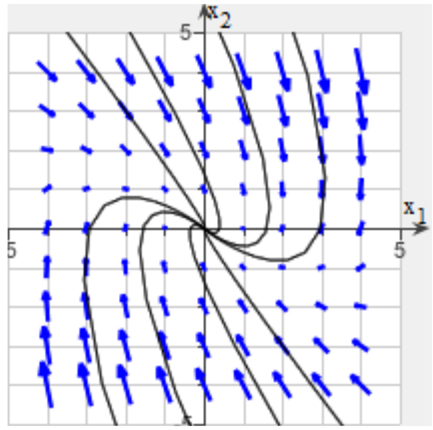


Improper Nodal Source: Distinct positive real eigenvalues (left) or a repeated positive real eigenvalue without two linearly independent eigenvectors (right).

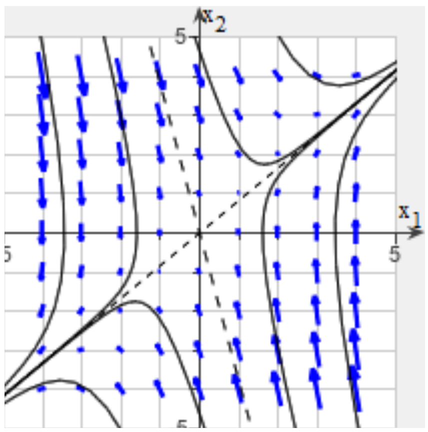
2: Definition

Gallery of Typical Phase Portraits for the System x'=Ax: Nodes

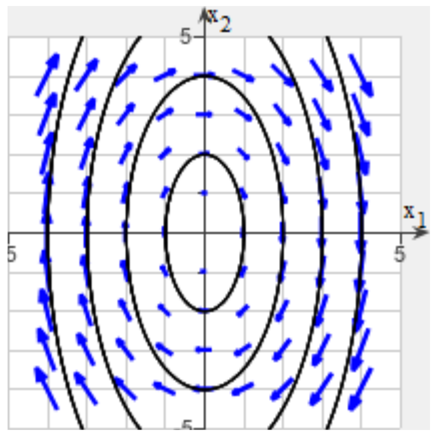




Improper Nodal Sink: Distinct negative real eigenvalues (left) or a repeated negative real eigenvalue without two linearly independent eigenvectors (right).



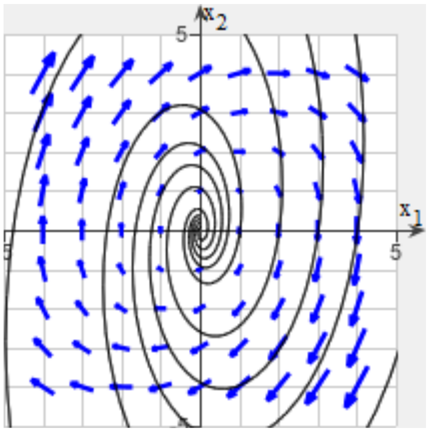
Saddle Point: Real eigenvalues of opposite sign.



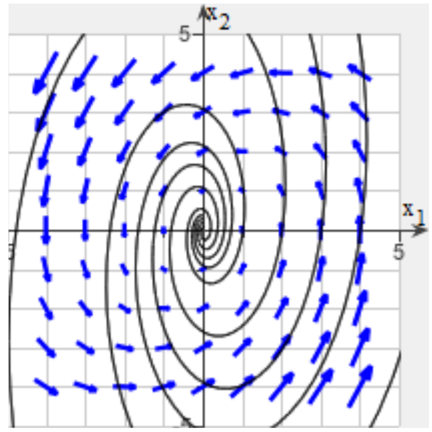
Center: Pure imaginary eigenvalues.

3: Definition

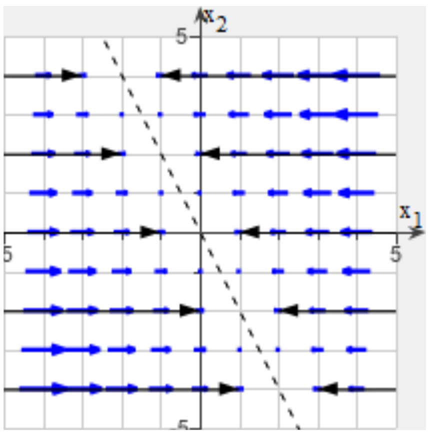
Gallery of Typical Phase Portraits for the System x'=Ax: Nodes



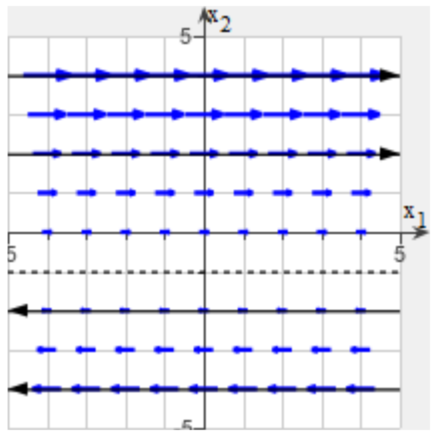
Spiral Source: Complex conjugate eigenvalues with positive real part.



Spiral Sink: Complex conjugate eigenvalues with negative real part.



Parallel Lines: One zero and one negative real eigenvalue. (If the nonzero eigenvalue is positive, then the trajectories flow away from the dotted line.)



Parallel Lines: A repeated zero eigenvalue without two linearly independent eigenvectors.

(1)
- ○ a proper nodal sink
- ○ an improper nodal source
- ◉ **a saddle point**
- ○ an improper nodal sink
- ○ a spiral sink
- ○ parallel lines
- ○ a center
- ○ a spiral source
- ○ a proper nodal source

(2)
- ○ distinct, negative, and real,
- ○ repeated, real, and zero,
- ○ complex with a negative real part,
- ○ distinct, real, with one zero,
- ○ complex with a positive real part,
- ◉ **distinct, opposite in sign, and real,**
- ○ purely imaginary,
- ○ distinct, positive, and real,
- ○ repeated, positive, and real,
- ○ repeated, negative, and real,

(3) ○ about which nothing else can be inferred.

○ that are approximately $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 5 \end{bmatrix}$.

◉ **that are approximately $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 5 \end{bmatrix}$.**

○ that are approximately $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

---

2. Transform the given system of differential equations into an equivalent system of first-order differential equations.

$x'' + 5x' + 5x + 2y = 0$
$y'' + 3y' + 2x - 2y = \textbf{sin } t$

Transform the given system of differential equations into an equivalent system of first-order differential equations.

$x'' - 4x' + 5x - 2y = 0$
$y'' - 2y' + 5x - 5y = \textbf{cos } t$

Let $x_1 = x$, $x_2 = x'$, $y_1 = y$, and $y_2 = y'$. Complete the system below.

$x_1' =$ $\mathbf{x_2}$
$x_2' =$ $\mathbf{-5x_2 - 5x_1 - 2y_1}$
$y_1' =$ $\mathbf{y_2}$
$y_2' =$ $\mathbf{-3y_2 - 2x_1 + 2y_1 + \sin t}$

Let . Complete the system below.

x@Sub{2}

@PRIME{x@Sub{2}}
=
4x@SUB{2}-5x@SUB{1}+2y@SUB{1}
@PRIME{y@Sub{1}}
=
y@SUB{2}
@PRIME{y@Sub{2}}
=
2y@SUB{2}-5x@SUB{1}+5y@SUB{1}+cos t

---

3. Find the general solutions of the system.

$$\mathbf{x}' = \begin{bmatrix} 5 & 0 & 0 \\ -1 & 6 & 1 \\ 0 & 0 & 5 \end{bmatrix} \mathbf{x}$$

Find the general solutions of the system.

$$\mathbf{x}' = \begin{bmatrix} 2 & 0 & 0 \\ -2 & 4 & 2 \\ 0 & 0 & 2 \end{bmatrix} \mathbf{x}$$

$$\mathbf{x}(t) = \underline{C_1 e^{5t} \bullet \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + C_2 e^{5t} \bullet \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + C_3 e^{6t} \bullet \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}$$

C@SUB{1}e@SUP{2t}*@MATX{{1};{1};{0}}+C@SUB{2}e@SUP{2t}*@MATX{{1};{0};{1}}+C@SUB{3}e@SUP{4t}*@MAT

---

4. What can be said about the following statements?

I) If A and B are square matrices, and det(B) is not equal to zero and B^{-1} is the inverse of B, then $BAB^{-1} - \lambda I = B(A - \lambda I)B^{-1}$ and so the matrices A and BAB^{-1} have the same eigenvalues.

II) If A is a square matrix and A^T is the transpose of A, then $\det(A - \lambda I) = \det\left(A^T - \lambda I\right)$ and so A and A^T have the same eigenvalues.

III) If A is a square matrix and det(A) is not equal to zero. If A^{-1} is the inverse of A and if λ is an eigenvalue of A then $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}.

IV) If a 4x4 matrix A is defective, then it must have one eigenvalue of multiplicity three.

- ○ **A.** I and III are true. II and IV are false
- ○ **B.** I, II and IV are true. III is false
- ○ **C.** I, III and IV are true. II is false
- ◉ **D. I, II and III are true. IV is false**
- ○ **E.** I and II are true. III and IV are false

What can be said about the following statements?

I) If A and B are square matrices, and det(B) is not equal to zero and is the inverse of B, then and so the matrices A and have the same eigenvalues.

II) If A is a square matrix and is the transpose of A, then and so A and have the same eigenvalues.

III) If A is a square matrix and det(A) is not equal to zero. If is the inverse of A and if is an eigenvalue of A then is an eigenvalue of .

IV) If a 4x4 matrix A is defective, then it must have one eigenvalue of multiplicity three.

I, II and III are true. IV is false
I and III are true. II and IV are false
I, II and IV are true. III is false
I and II are true. III and IV are false
I, III and IV are true. II is false

5. Let y(x) satisfy the following initial value problem:

$y''(x) + y(x) = \tan(x)$

$y(0) = 0$ and $y'(0) = -1$

Then $y\left(\frac{\pi}{4}\right)$ (which is the value of y(x) when $x = \frac{\pi}{4}$) is equal to:

- ○ **A.** $y\left(\frac{\pi}{4}\right) = \sqrt{2} - \frac{\sqrt{2}}{2} \ln\left(1 + \sqrt{2}\right)$
- ◉ **B.** $y\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} \ln\left(1 + \sqrt{2}\right)$
- ○ **C.** $y\left(\frac{\pi}{4}\right) = -3\sqrt{2} - \sqrt{2} \ln\left(1 + \sqrt{2}\right)$
- ○ **D.** $y\left(\frac{\pi}{4}\right) = \sqrt{2} + \frac{\sqrt{2}}{2} \ln\left(1 + \sqrt{2}\right)$
- ○ **E.** $y\left(\frac{\pi}{4}\right) = 2\sqrt{2} - \ln\left(1 + \sqrt{2}\right)$

Let y(x) satisfy the following initial value problem:

Then (which is the value of y(x) when) is equal to:

y(@DIV{π;4})= -@DIV{@RT{2};2} ln(1+@RT{2})
y(@DIV{π;4})= 2@RT{2}-ln(1+@RT{2})
y(@DIV{π;4})= -3@RT{2}- @RT{2}ln(1+@RT{2})
y(@DIV{π;4})=@RT{2} + @DIV{@RT{2};2}ln(1+@RT{2})
y(@DIV{π;4})= @RT{2} - @DIV{@RT{2};2} ln(1+ @RT{2})

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6. Categorize the eigenvalues and eigenvectors of the coefficient matrix **A** according to the accompanying classifications and sketch the phase portrait of the system by hand. Then use a computer system or graphing calculator to check your answer.

System of equations	Matrix equation
$x_1{}' = 5x_1 + 7x_2$ $x_2{}' = 7x_1 + 5x_2$	$\mathbf{x}' = \begin{bmatrix} 5 & 7 \\ 7 & 5 \end{bmatrix}\mathbf{x}$
Eigenvalues	**Eigenvectors**
$\lambda_1 = -2, \lambda_2 = 12$	$\mathbf{v}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

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Categorize the eigenvalues and eigenvectors of the coefficient matrix **A** according to the accompanying classifications and sketch the phase portrait of the system by hand. Then use a computer system or graphing calculator to check your answer.

System of equations	Matrix equation
$x_1{}' = 2x_1 + 6x_2$ $x_2{}' = 6x_1 + 2x_2$	$\mathbf{x}' = \begin{bmatrix} 2 & 6 \\ 6 & 2 \end{bmatrix}\mathbf{x}$
Eigenvalues	**Eigenvectors**
$\lambda_1 = -4, \lambda_2 = 8$	$\mathbf{v}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

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Click here to view page 2 of Gallery of Typical Phase Portraits for the System x'=Ax: Nodes
Click here to view page 3 of Gallery of Typical Phase Portraits for the System x'=Ax: Nodes

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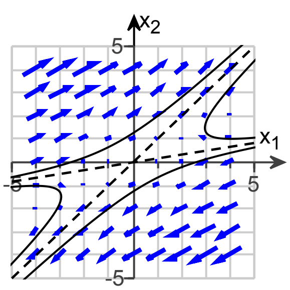
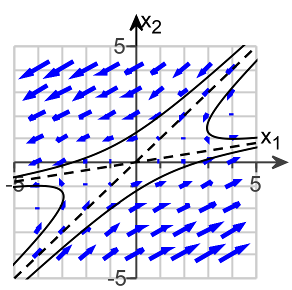
The system shows (1) ______________ and its eigenvalues are (2) ______________

Sketch a graph of the phase portrait. Choose the correct answer below.

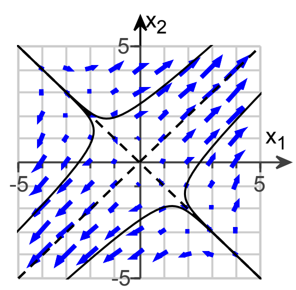
A.



B.



C.



an improper nodal source
parallel lines
a proper nodal sink
an improper nodal sink
a proper nodal source
a center
a spiral source
a spiral sink
a saddle point

The system shows and its eigenvalues are

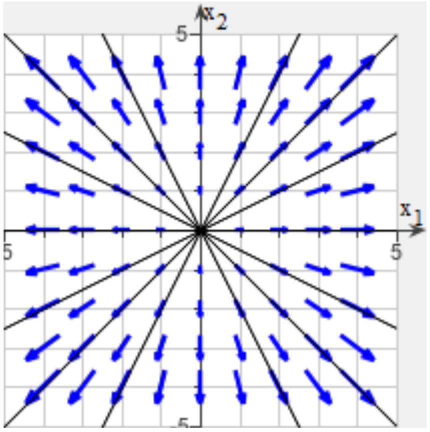
distinct, positive, and real.
complex with a positive real part.
distinct, real, with one zero.
distinct, opposite in sign, and real.
repeated, real, and zero.
complex with a negative real part.
purely imaginary.
repeated, positive, and real.

repeated, negative, and real.

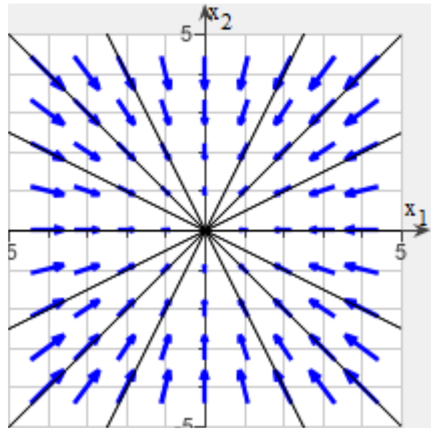
Sketch a graph of the phase portrait. Choose the correct answer below.

4: Test

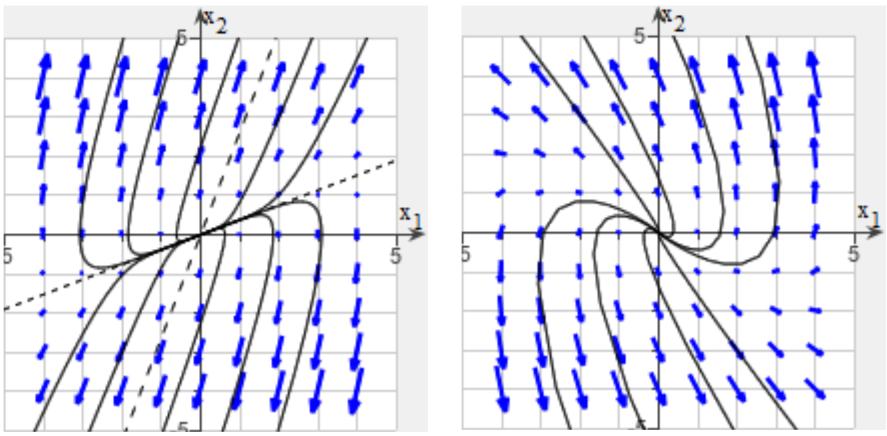
Gallery of Typical Phase Portraits for the System x'=Ax: Nodes



Proper Nodal Source: A repeated positive real eigenvalue with two linearly independent eigenvectors.



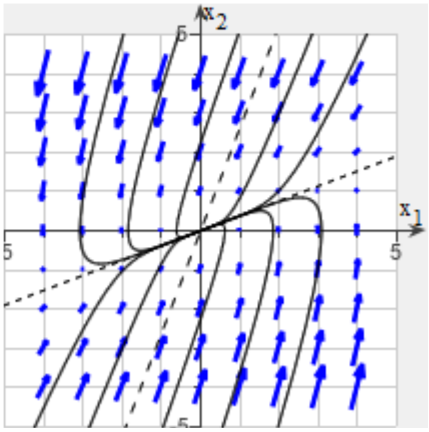
Proper Nodal Sink: A repeated negative real eigenvalue with two linearly independent eigenvectors.

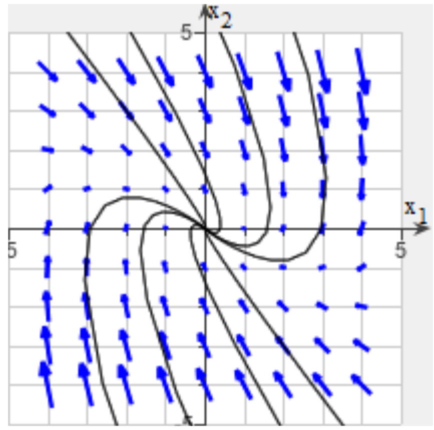


Improper Nodal Source: Distinct positive real eigenvalues (left) or a repeated positive real eigenvalue without two linearly independent eigenvectors (right).

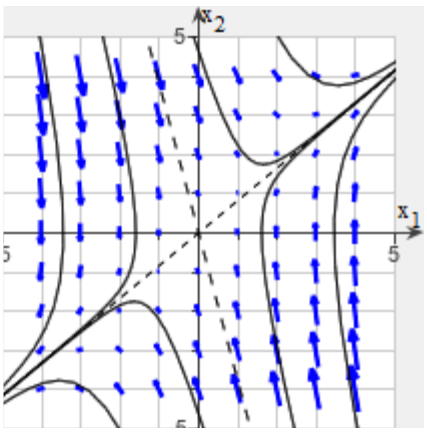
5: Definition

Gallery of Typical Phase Portraits for the System x'=Ax: Nodes

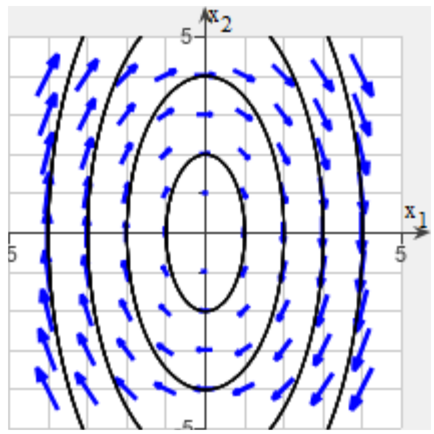




Improper Nodal Sink: Distinct negative real eigenvalues (left) or a repeated negative real eigenvalue without two linearly independent eigenvectors (right).



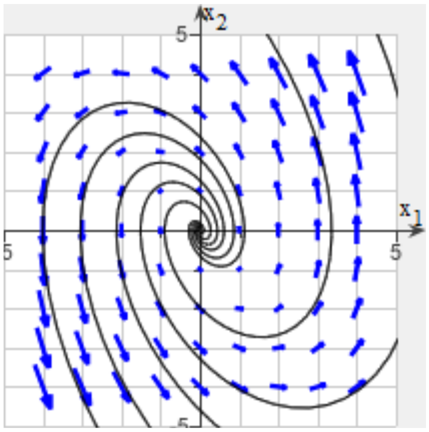
Saddle Point: Real eigenvalues of opposite sign.



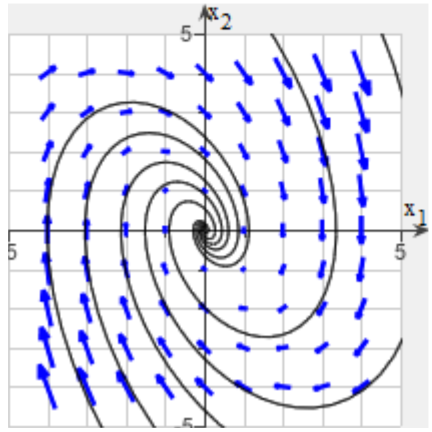
Center: Pure imaginary eigenvalues.

6: Definition

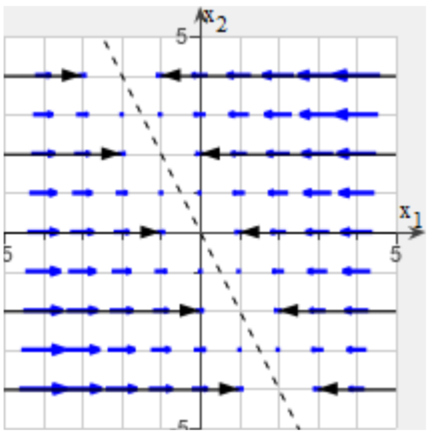
Gallery of Typical Phase Portraits for the System x'=Ax: Nodes



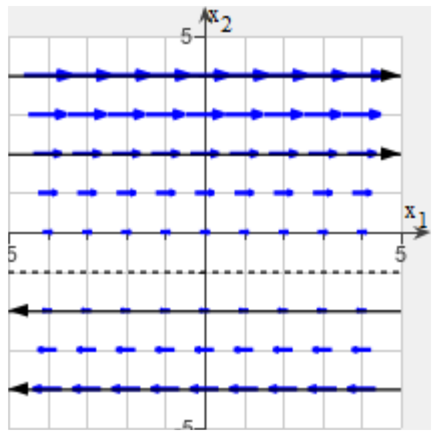
Spiral Source: Complex conjugate eigenvalues with positive real part.



Spiral Sink: Complex conjugate eigenvalues with negative real part.



Parallel Lines: One zero and one negative real eigenvalue. (If the nonzero eigenvalue is positive, then the trajectories flow away from the dotted line.)

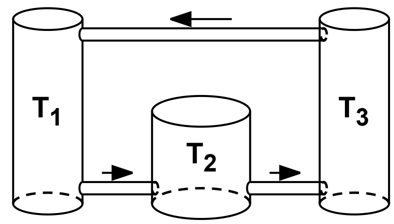


Parallel Lines: A repeated zero eigenvalue without two linearly independent eigenvectors.

(1)
- ○ a proper nodal source
- ○ a spiral source
- ○ an improper nodal source
- ○ a spiral sink
- ○ a proper nodal sink
- ◉ **a saddle point**
- ○ parallel lines
- ○ a center
- ○ an improper nodal sink

(2)
- ○ complex with a negative real part.
- ○ distinct, real, with one zero.
- ○ distinct, positive, and real.
- ◉ **distinct, opposite in sign, and real.**
- ○ purely imaginary.
- ○ repeated, positive, and real.
- ○ repeated, negative, and real.
- ○ repeated, real, and zero.
- ○ complex with a positive real part.

7. Three 234-gal fermentation vats are connected as indicated in the figure, and the mixtures in each tank are kept uniform by stirring. Denote by $x_i(t)$ the amount (in pounds) of alcohol in tank T_i at time t ($i = 1, 2, 3$). Suppose that the mixture circulates between the tanks at the rate of 18 gal/min. Derive the equations.



$$13x_1' = -x_1 + x_3$$
$$13x_2' = x_1 - x_2$$
$$13x_3' = x_2 - x_3$$

165
Three -gal fermentation vats are connected as indicated in the figure, and the mixtures in each tank are kept uniform by stirring.
x@Sub{i}
Denote by (t) the amount (in pounds) of alcohol in tank
(i
T@Sub{i} =
at time t 1, 2, 3). Suppose that the mixture circulates
15
between the tanks at the rate of gal/min. Derive the equations.

11@PRIME{x@Sub{1}}= - x@Sub{1} +x@sub{3}
11@PRIME{x@Sub{2}}=x@Sub{1} - x@sub{2}
11@PRIME{x@Sub{3}}= x@Sub{2}-x@sub{3}

Calculate the concentration of alcohol in each tank.

The alcohol concentration in tank T_1 is $\frac{x_1}{234}$ (1) ______

The alcohol concentration in tank T_2 is $\frac{x_2}{234}$ (2) ______

The alcohol concentration in tank T_3 is $\frac{x_3}{234}$ (3) ______

Calculate the rate of change of the amount of alcohol in each tank.

$x_1' = -\frac{x_1}{13} + \frac{x_3}{13}$ (4) ______

$x_2' = \frac{x_1}{13} - \frac{x_2}{13}$ (5) ______

$x_3' = \frac{x_2}{13} - \frac{x_3}{13}$ (6) ______

What final step is needed to obtain the derived equations given in the problem statement?

Multiply both sides of the first equation by **13**.
Multiply both sides of the second equation by **13**.
Multiply both sides of the third equation by **13**.

Calculate the concentration of alcohol in each tank.

@DIV{x@SUB{1};165}
The alcohol concentration in tank is

@div{gallons;pound}.
pounds.
gallons.
@div{pounds;gallon}.

The alcohol concentration in tank T@Sub{2} is **@DIV{x@SUB{2};165}**
gallons.
@div{gallons;pound}.
pounds.
@div{pounds;gallon}.

The alcohol concentration in tank T@Sub{3} is **@DIV{x@SUB{3};165}**
@div{gallons;pound}.
pounds.
gallons.
@div{pounds;gallon}.

Calculate the rate of change of the amount of alcohol in each tank.

-@DIV{x@SUB{1};11}+@DIV{x@SUB{3};11}

@div{minutes;pound}
@div{pounds;minute}
@div{gallons;minute}
@div{minutes;gallon}
@PRIME{x@Sub{2}}
=
@DIV{x@SUB{1};11}-@DIV{x@SUB{2};11}

@div{pounds;minute}
@div{gallons;minute}
@div{minutes;pound}
@div{minutes;gallon}
@PRIME{x@Sub{3}}
=
@DIV{x@SUB{2};11}-@DIV{x@SUB{3};11}

@div{minutes;pound}
@div{pounds;minute}
@div{minutes;gallon}
@div{gallons;minute}

What final step is needed to obtain the derived equations given in the problem statement?

Multiply both sides of the first equation by **11**.
Multiply both sides of the second equation by **11**.
Multiply both sides of the third equation by **11**.

(1)
- ○ pounds.
- ○ $\frac{\text{gallons}}{\text{pound}}$.
- ○ gallons.
- ◉ $\frac{\textbf{pounds}}{\textbf{gallon}}$.

(2)
- ○ gallons.
- ○ $\frac{\text{gallons}}{\text{pound}}$.
- ○ pounds.
- ◉ $\frac{\textbf{pounds}}{\textbf{gallon}}$.

(3)
- ○ pounds.
- ○ gallons.
- ◉ $\frac{\textbf{pounds}}{\textbf{gallon}}$.
- ○ $\frac{\text{gallons}}{\text{pound}}$.

(4)
- ○ $\frac{\text{gallons}}{\text{minute}}$
- ◉ $\frac{\textbf{pounds}}{\textbf{minute}}$
- ○ $\frac{\text{minutes}}{\text{pound}}$
- ○ $\frac{\text{minutes}}{\text{gallon}}$

(5)
- ◉ $\frac{\textbf{pounds}}{\textbf{minute}}$
- ○ $\frac{\text{gallons}}{\text{minute}}$
- ○ $\frac{\text{minutes}}{\text{gallon}}$
- ○ $\frac{\text{minutes}}{\text{pound}}$

(6)
- ○ $\frac{\text{gallons}}{\text{minute}}$
- ○ $\frac{\text{minutes}}{\text{pound}}$
- ○ $\frac{\text{minutes}}{\text{gallon}}$
- ◉ $\frac{\textbf{pounds}}{\textbf{minute}}$

---

8. Let y(t) be the solution of the following equalton representing a spring-mass system:

$y''(t) + 4y'(t) + 5y(t) = 0$

$y(0) = A$ and $y'(0) = B$

with $A \neq 0$ and $B \neq 0$. Then $\frac{y(\pi)}{y(3\pi)}$ (this is the quotient of the values of $y(\pi)$ and $y(3\pi)$) is equal to.

- ○ **A.** $e^{4\pi}\frac{A}{B}$
- ◉ **B.** $e^{\mathbf{4\pi}}$
- ○ **C.** $e^{-4\pi}$
- ○ **D.** $e^{4\pi}\frac{B}{A}$
- ○ **E.** $e^{\pi}A + e^{3\pi}B$

Let be the solution of the following equalton representing a spring-mass system:

with Then (this is the quotient of the values of and) is equal to.

e@Sup{-4π}
e@Sup{4π}@DIV{ A;B}
e@Sup{π}A + e@Sup{3π}B
e@Sup{4π}@DIV{ B;A}
e@Sup{4π}

---

9. The appropriate form of a particular solution of the differential equation

$(D-1)^3(D-3)^4\left(D^2+1\right)\,y(x) = x^3\,e^{x} + x^4\,e^{3x} + x^2\,\sin(x)$

is of the form

$y_p(x) = x^3 p_1(x)\,e^{x} + x^4\,p_2(x)\,e^{3x} + x\,p_3(x)\,\sin(x) + x\,p_4(x)\,\cos(x)$,

where $p_1(x)$ is a polynomial of degree d_1, $p_2(x)$ is a polynomial of degree d_2, $p_3(x)$ is a polynomial of degree d_3, and $p_4(x)$ is a polynomial of dgree d_4. Which of the following is true?

- **A.** $d_1=1$, $d_2=3$, $d_3=2$ and $d_4=2$
- **B.** $d_1=3$, $d_2=3$, $d_3=1$ and $d_4=1$
- **C.** $d_1=2$, $d_2=2$, $d_3=1$ and $d_4=1$
- **D.** **$d_1=3$, $d_2=4$, $d_3=2$ and $d_4=2$**
- **E.** $d_1=3$, $d_2=4$, $d_3=1$ and $d_4=1$

The appropriate form of a particular solution of the differential equation

is of the form

,

where is a polynomial of degree , is a polynomial of degree , is a polynomial of degree , and is a polynomial of dgree . Which of the following is true?

d@Sub{1}=3, d@Sub{2}=3, d@Sub{3}=1 and d@Sub{4}=1
d@Sub{1}=2, d@Sub{2}=2, d@Sub{3}=1 and d@Sub{4}=1
d@Sub{1}=1, d@Sub{2}= 3, d@Sub{3}=2 and d@Sub{4}=2
d@Sub{1}=3, d@Sub{2}=4, d@Sub{3}=1 and d@Sub{4}=1
d@Sub{1}=3, d@Sub{2}=4, d@Sub{3}=2 and d@Sub{4}=2

10. Find the general solution of the given system. Use a computer system or graphing calculator to construct a direction field and typical solution curves for the system.

$$\mathbf{x}' = \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}\mathbf{x}$$

Find the general solution of the given system. Use a computer system or graphing calculator to construct a direction field and typical solution curves for the system.

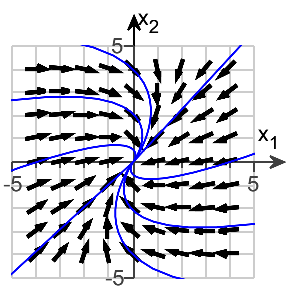
$$\mathbf{x}' = \begin{bmatrix} 5 & 1 \\ -1 & 3 \end{bmatrix}\mathbf{x}$$

What is the general solution to the system?

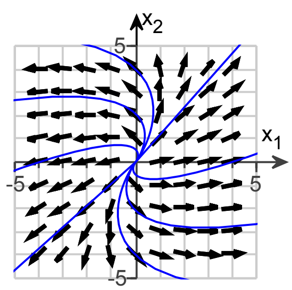
$$\mathbf{x}(t) = C_1 e^{2t} \cdot \begin{bmatrix} 1 \\ -1 \end{bmatrix} + C_2 e^{2t} \cdot \begin{bmatrix} t \\ -t+1 \end{bmatrix}$$

Graph the direction field with several solution curves. Choose the correct graph below.

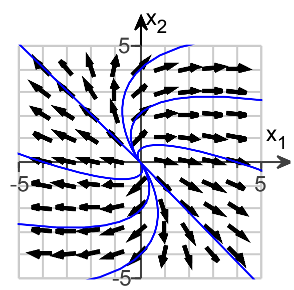
A.



B.



C.



What is the general solution to the system?

C@SUB{1}e@SUP{4t}*@MATX{{1};{-1}}+C@SUB{2}e@SUP{4t}*@MATX{{t};{-t+1}}

Graph the direction field with several solution curves. Choose the correct graph below.

11. Apply the eigenvalue method to find a general solution of the given system. Find the particular solution corresponding to the given initial values. Use a computer system or graphing calculator to construct a direction field and typical solution curves for the given system.

$$x'_1 = 3x_1 + 4x_2,\ x'_2 = 3x_1 + 2x_2,\ x_1(0) = x_2(0) = 1$$

Apply the eigenvalue method to find a general solution of the given system. Find the particular solution corresponding to the given initial values. Use a computer system or graphing calculator to construct a direction field and typical solution curves for the given system.

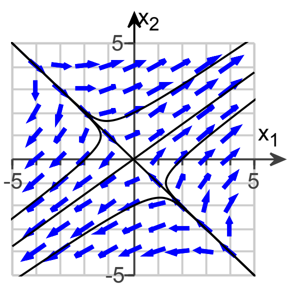
$$x'_1 = 3x_1 + 4x_2,\ x'_2 = 3x_1 + 2x_2,\ x_1(0) = x_2(0) = 1$$

The general solution in matrix form is $\mathbf{x}(t) = \begin{bmatrix} 4c_1 e^{6t} + c_2 e^{-t} \\ 3c_1 e^{6t} - c_2 e^{-t} \end{bmatrix}$.

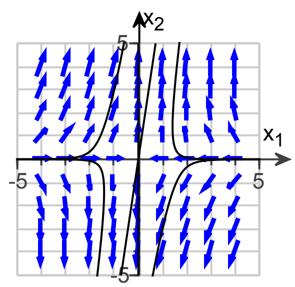
The particular solution in matrix form is $\mathbf{x}(t) = \begin{bmatrix} \frac{8}{7} e^{6t} - \frac{1}{7} e^{-t} \\ \frac{6}{7} e^{6t} + \frac{1}{7} e^{-t} \end{bmatrix}$.

Choose the correct graph below.

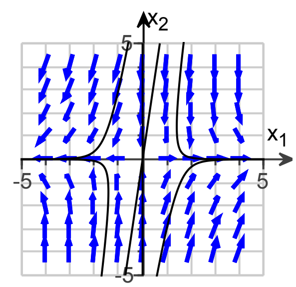
(selected) **A.**



B.



C.



The general solution in matrix form is

@MATX{{4c@Sub{1}e@Sup{6t}+c@Sub{2}e@Sup{-t}};{3c@Sub{1}e@Sup{6t}-c@Sub{2}e@Sup{-t}}}

.

The particular solution in matrix form is

@MATX{{@DIV{8;7}e@SUP{6t}-@DIV{1;7}e@SUP{-t}};{@DIV{6;7}e@SUP{6t}+@DIV{1;7}e@SUP{-t}}}

.

Choose the correct graph below.

12. Apply the eigenvalue method to find a general solution of the given system. Use a computer system or graphing calculator to construct a direction field and typical solution curves for the given system.

$$x'_1 = 10x_1 - 10x_2,\ x'_2 = 8x_1 + 2x_2$$

Apply the eigenvalue method to find a general solution of the given system. Use a computer system or graphing calculator to construct a direction field and typical solution curves for the given system.

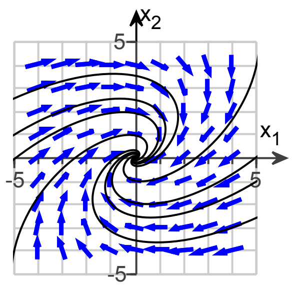
$$x'_1 = 6x_1 - 5x_2,\ x'_2 = 4x_1 + 2x_2$$

What is the general solution in matrix form?

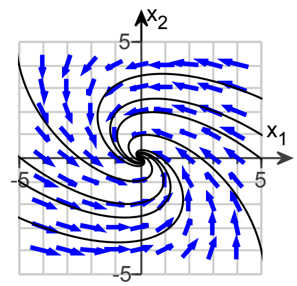
$$\mathbf{x}(t) = \begin{bmatrix} c_1 e^{6t}(\cos 8t - 2\sin 8t) + c_2 e^{6t}(2\cos 8t + \sin 8t) \\ c_1 e^{6t}(2\cos 8t) + c_2 e^{6t}(2\sin 8t) \end{bmatrix}$$

Choose the correct graph below.

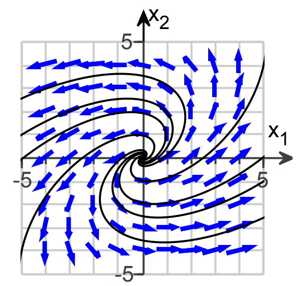
A.



B.



C. (selected)



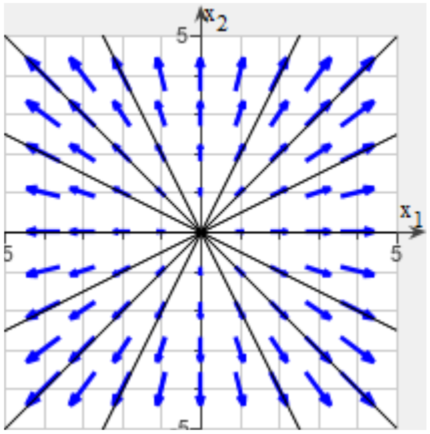
What is the general solution in matrix form?

@MATX{{c@Sub{1}e@Sup{4t}(cos4t-2sin4t)+c@Sub{2}e@Sup{4t}(2cos4t+sin4t)};{c@Sub{1}e@Sup{4t}(2cos4t)+c@

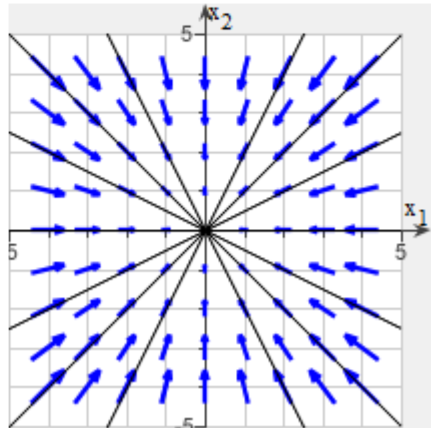
Choose the correct graph below.

7: Test

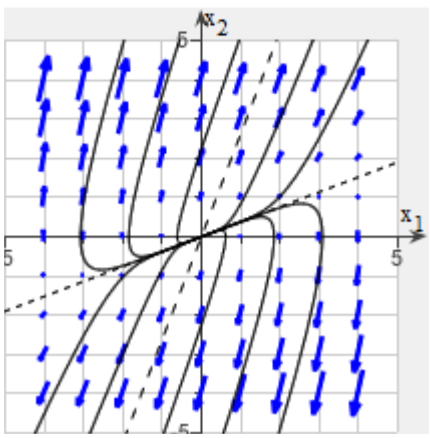
Gallery of Typical Phase Portraits for the System x'=Ax: Nodes

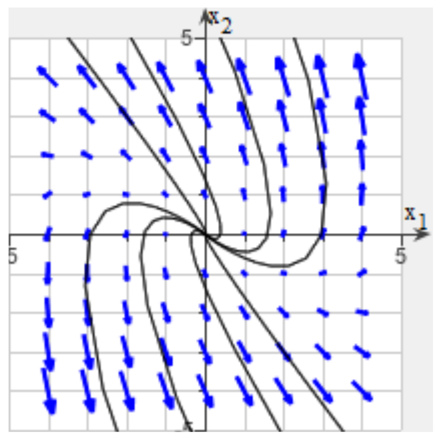


Proper Nodal Source: A repeated positive real eigenvalue with two linearly independent eigenvectors.



Proper Nodal Sink: A repeated negative real eigenvalue with two linearly independent eigenvectors.

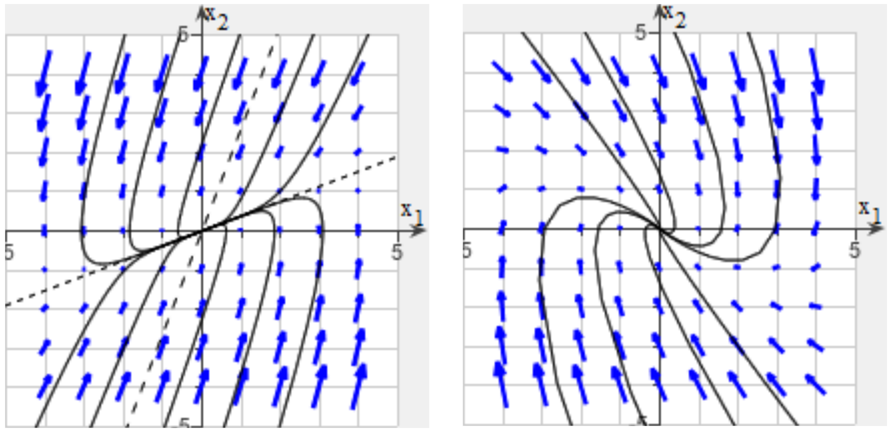




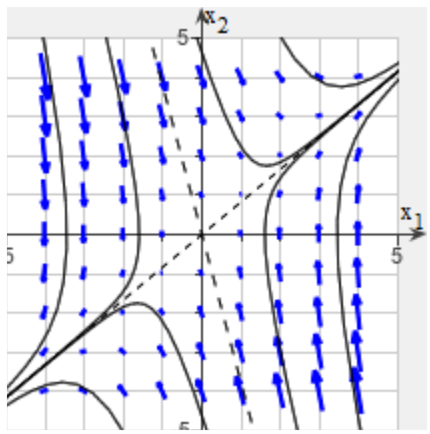
Improper Nodal Source: Distinct positive real eigenvalues (left) or a repeated positive real eigenvalue without two linearly independent eigenvectors (right).

8: Definition

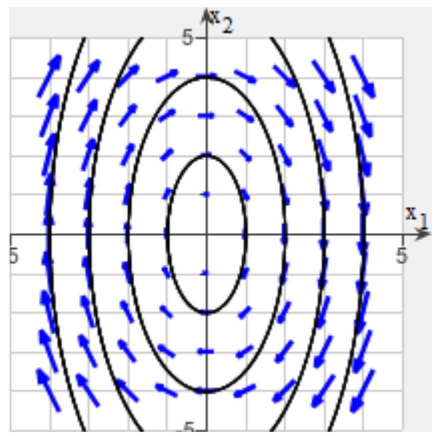
Gallery of Typical Phase Portraits for the System x'=Ax: Nodes



Improper Nodal Sink: Distinct negative real eigenvalues (left) or a repeated negative real eigenvalue without two linearly independent eigenvectors (right).



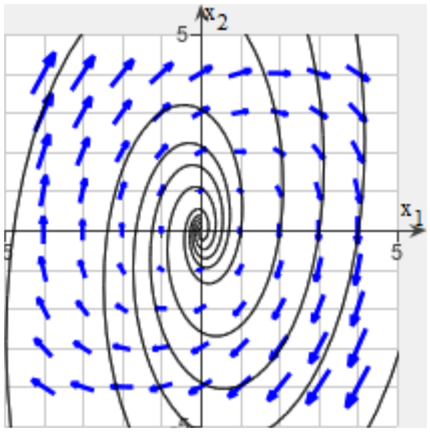
Saddle Point: Real eigenvalues of opposite sign.



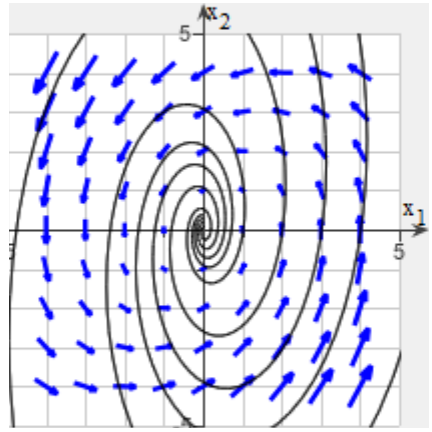
Center: Pure imaginary eigenvalues.

9: Definition

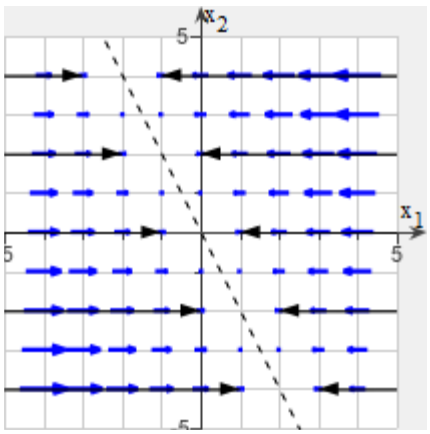
Gallery of Typical Phase Portraits for the System x'=Ax: Nodes



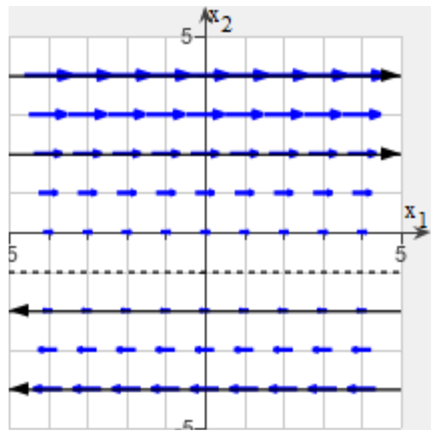
Spiral Source: Complex conjugate eigenvalues with positive real part.



Spiral Sink: Complex conjugate eigenvalues with negative real part.



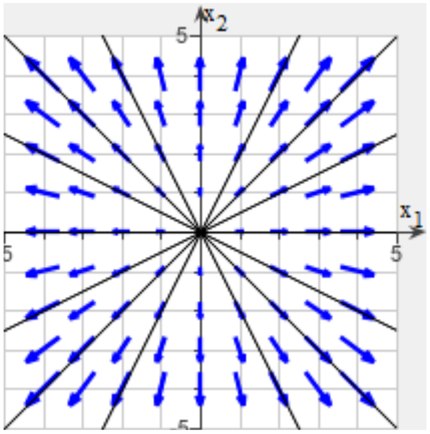
Parallel Lines: One zero and one negative real eigenvalue. (If the nonzero eigenvalue is positive, then the trajectories flow away from the dotted line.)



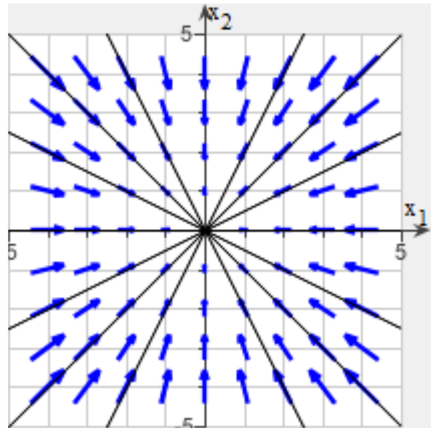
Parallel Lines: A repeated zero eigenvalue without two linearly independent eigenvectors.

10: Test

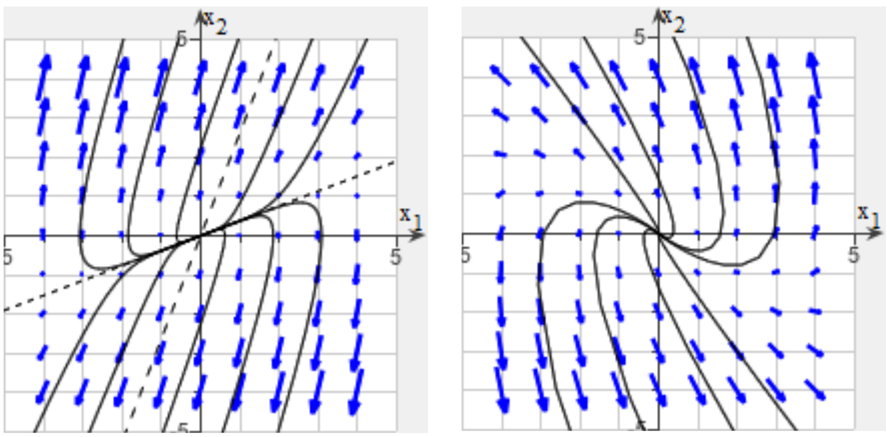
Gallery of Typical Phase Portraits for the System x'=Ax: Nodes



Proper Nodal Source: A repeated positive real eigenvalue with two linearly independent eigenvectors.



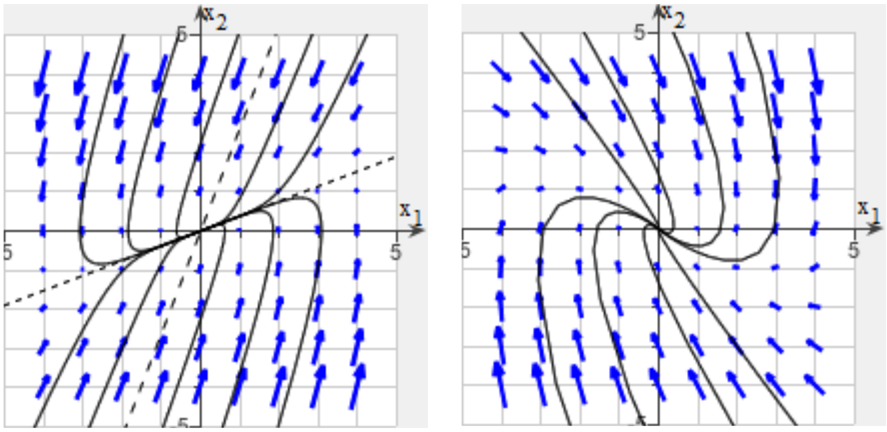
Proper Nodal Sink: A repeated negative real eigenvalue with two linearly independent eigenvectors.



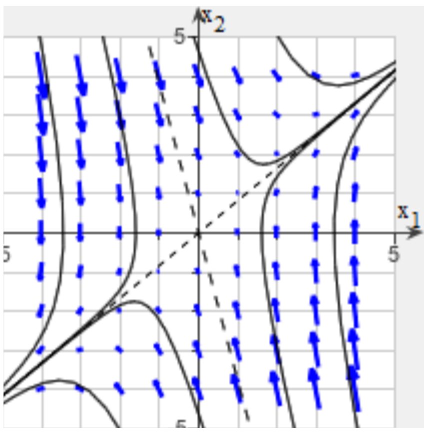
Improper Nodal Source: Distinct positive real eigenvalues (left) or a repeated positive real eigenvalue without two linearly independent eigenvectors (right).

11: Definition

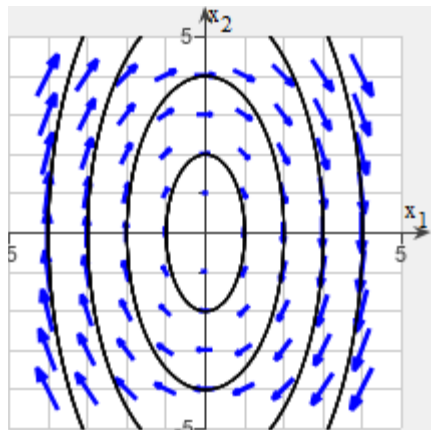
Gallery of Typical Phase Portraits for the System x'=Ax: Nodes



Improper Nodal Sink: Distinct negative real eigenvalues (left) or a repeated negative real eigenvalue without two linearly independent eigenvectors (right).



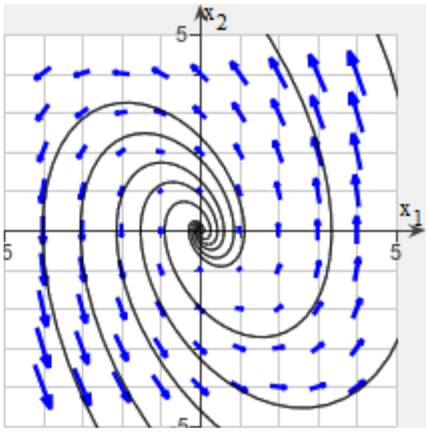
Saddle Point: Real eigenvalues of opposite sign.



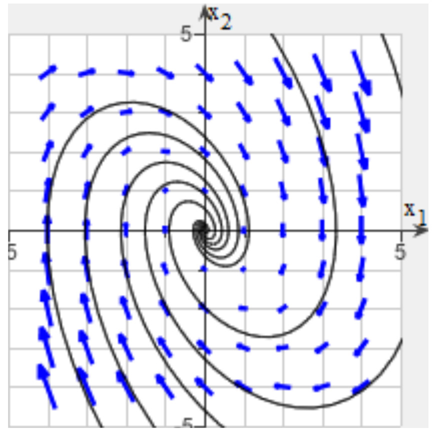
Center: Pure imaginary eigenvalues.

12: Definition

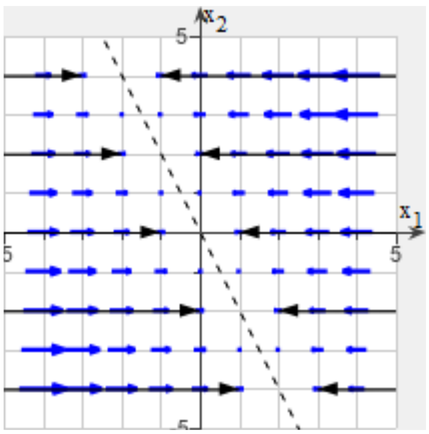
Gallery of Typical Phase Portraits for the System x'=Ax: Nodes



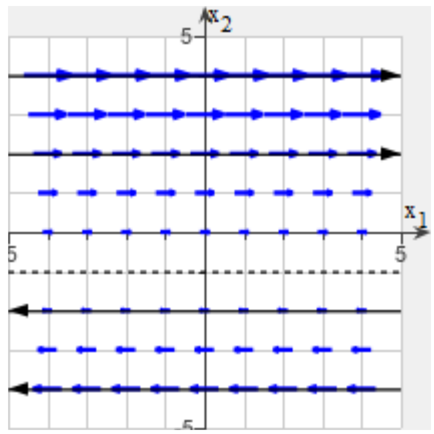
Spiral Source: Complex conjugate eigenvalues with positive real part.



Spiral Sink: Complex conjugate eigenvalues with negative real part.



Parallel Lines: One zero and one negative real eigenvalue. (If the nonzero eigenvalue is positive, then the trajectories flow away from the dotted line.)



Parallel Lines: A repeated zero eigenvalue without two linearly independent eigenvectors.