MA262 — FINAL EXAM — FALL 2019 — DECEMBER 11, 2019
TEST NUMBER 11 — GREEN

INSTRUCTIONS:

1. Do not open the exam booklet until you are instructed to do so.
2. Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.
3. *Mark your test number on your scantron*
4. Once you are allowed to open the exam, make sure you have a complete test. There are 14 different test pages including this cover page.
5. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers on this test booklet.
6. The exam has 25 problems and each one is worth 8 points. The maximum possible score is 200 points. No partial credit.
7. Do not leave the exam room during the first 20 minutes of the exam.
8. If you do not finish your exam in the first 100 minutes, you must wait until the end of the exam period to leave the room.
9. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

DON'T BE A CHEATER:

1. Do not give, seek or obtain any kind of help from anyone to answer questions on this exam. If you have doubts, consult only your instructor.
2. Do not look at the exam or scantron of another student.
3. Do not allow other students to look at your exam or your scantron.
4. You may not compare answers with anyone else or consult another student until after you have finished your exam, given it to your instructor and left the room.
5. Do not consult notes or books.
6. Do not handle phones or cameras, calculators or any electronic device until after you have finished your exam, given it to your instructor and left the room.
7. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs collect the scantrons and the exams.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty include an F in the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

STUDENT ID NUMBER: ______________________________

SECTION NUMBER AND RECITATION INSTRUCTOR: ______________________________

1. If

$$\begin{pmatrix} 1 & 5 & 3 \\ 4 & 2 & 6 \end{pmatrix} \begin{pmatrix} a & 1 \\ 0 & b \\ -b & -a \end{pmatrix} = \begin{pmatrix} 7 & -12 \\ 16 & -6 \end{pmatrix},$$

then

A. $a = 6, b = -\frac{1}{3}$

B. $a = 10, b = 1$

C. $a = 4, b = -1$

D. $a = 7, b = 0$

E. $a = 1, b = -2$

2. If the system

$$\begin{aligned} 3x + y - 5z &= a \\ 2x + 2y - 3z &= b \\ x - y - 2z &= c \end{aligned}$$

is consistent, what can we conclude about a, b and c?

A. $c^2 = a^2$

B. $a + b = 6$

C. $c = 3$

D. $c = a - b$

E. $c = a + b$

3. Find all the values of a such that

$$A = \begin{bmatrix} 1 & 1-a & 0 \\ 1 & 0 & 0 \\ 1 & 2 & a \end{bmatrix}$$

is invertible.

A. $a \neq 0$ and $a \neq 1$.

B. $a \neq 2$.

C. $a \neq -1$.

D. It is invertible for all values of a.

E. $a \neq 0$

4. Which of the following statements about all 5×5 matrices is **true**?

A. $\det(A+B) = \det A + \det B$

B. $\det A^T = -\det(A)$.

C. $AB = 0$ implies $A = 0$ or $B = 0$.

D. If $\det A = 0$ then some two rows are proportional.

E. $\det(-A) = -\det(A)$.

5. Let V be the set of positive real numbers. Let the definition of vector addition be $\mathbf{u} \oplus \mathbf{v} = e^{\mathbf{u}} e^{\mathbf{v}}$ for every $\mathbf{u}$ and $\mathbf{v}$ in V and the definition of multiplication by a scalar be $c \odot \mathbf{u} = e^{c\mathbf{u}}$ for every real number c and every $\mathbf{u}$ in V. Which statement below is **not** true?

A. V is closed under vector addition.

B. V is closed under scalar multiplication.

C. The vector addition is commutative: $\mathbf{u} \oplus \mathbf{v} = \mathbf{v} \oplus \mathbf{u}$ for every $\mathbf{u}$ and $\mathbf{v}$ in V.

D. It is true that $1 \odot \mathbf{u} = \mathbf{u}$ for every $\mathbf{u}$ in V.

E. It is true that $0 \odot \mathbf{u} = 1$ for every $\mathbf{u}$ in V.

6. Consider the real vector space $\mathbb{M}_2$ of all real 2×2 matrices. Let B be a fixed matrix in $\mathbb{M}_2$. Which of the following sets is **not** a subspace of $\mathbb{M}_2$?

A. The set of all the matrices, A in $\mathbb{M}_2$ such that $AB = BA$.

B. The set of all the matrices, A in $\mathbb{M}_2$ such that $AB = O$, where O is the zero matrix in $\mathbb{M}_2$.

C. The set of all the matrices, A in $\mathbb{M}_2$ such that $A^2 = O$, where O is the zero matrix in $\mathbb{M}_2$.

D. All the upper triangular matrices in $\mathbb{M}_2$.

E. All the symmetric matrices in $\mathbb{M}_2$.

7. What is the dimension of the space span $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$?

A. 0

B. 1

C. 2

D. 3

E. 4

8. Let $A = \begin{pmatrix} 2 & -1 & -2 \\ -4 & 2 & 4 \\ -8 & 4 & 8 \end{pmatrix}$. Then the following is a basis of the nullspace of A.

A. $\{[2\ -4\ -8]^T, [-1\ 2\ 4]^T, [-2\ 4\ 8]^T\}$

B. $\{[1\ 0\ 1]^T, [1\ 2\ 0]^T\}$

C. $\{[1\ 0\ 1]^T, [1\ 2\ 0]^T, [3\ 2\ 2]^T\}$

D. $\{[1\ 0\ 1]^T\}$

E. $\{[0\ 2\ 1]^T\}$

9. Let A be an $m \times n$ matrix. Then the linear system $A\mathbf{x} = \mathbf{b}$ has a solution for any $m \times 1$ matrix $\mathbf{b}$ if and only if

A. $m = n$.

B. Nullity(A)$= m$.

C. Rank(A)+Nullity(A)$= n$.

D. $A = I$ the identity matrix.

E. Rank(A)$= m$.

10. Let $L : \mathbb{R}_2 \to \mathbb{R}_2$ be a linear transformation for which we know that

$$L\left(\left[\begin{array}{cc} 1 & 1 \end{array}\right]\right) = \left[\begin{array}{cc} 1 & -2 \end{array}\right], \; L\left(\left[\begin{array}{cc} 1 & -1 \end{array}\right]\right) = \left[\begin{array}{cc} 2 & 1 \end{array}\right].$$

What is $L\left(\left[\begin{array}{cc} 3 & 1 \end{array}\right]\right)$?

A. $\left[\begin{array}{cc} 3 & 1 \end{array}\right]$

B. $\left[\begin{array}{cc} 1 & -2 \end{array}\right]$

C. $\left[\begin{array}{cc} 2 & 1 \end{array}\right]$

D. $\left[\begin{array}{cc} 4 & -3 \end{array}\right]$

E. $\left[\begin{array}{cc} 5 & -5 \end{array}\right]$

11. Which of the following statements is correct?

A. An eigenvector of a matrix may correspond to two distinct eigenvalues of the matrix.

B. Any 2×2 matrix must have two linearly independent eigenvectors.

C. If $\mathbf{x}$ is an eigenvector of a matrix A, then $c\mathbf{x}$ is also an eigenvector of A, where c is any nonzero scalar.

D. Any 2×2 matrix can not have complex eigenvalues.

E. If a 2×2 matrix A is similar to a diagonal matrix D, then D must be unique.

12. Which one of the following statements correctly describes eigenvalues of a real $n \times n$ symmetric matrix M?

A. All eigenvalues are real numbers.

B. Sometimes M can have no eigenvalues at all.

C. Some eigenvalues are real and some are not.

D. M has always n distinct real eigenvalues.

E. M has always n distinct complex eigenvalues.

13. The solution of $\dfrac{dy}{dx} - \dfrac{2}{x}y = x^2 - 1$ with $y(1) = 3$ is

A. $y = x^3 + x + 1$

B. $y = x^3 + x^2 + 1$

C. $y = x^3 + x^2 + x$

D. $y = x^3 - x^2 + 3x$

E. $y = x^4 + x^2 + 1$

14. An implicit solution of $y' = \dfrac{2x}{y + x^2 y}$ is

A. $y^2 = 2\ln(1 + x^2) + C$

B. $y^2 = C\ln(1 + x^2)$

C. $\dfrac{1}{2}y^2 = \ln x^2 + C$

D. $y^2 = \ln(1 + x^2) + C$

E. $\dfrac{1}{2}y^2 = \ln|1 + x| + C$

15. The substitution $v = \dfrac{y}{x}$ transforms the equation $\dfrac{dy}{dx} = \sin\left(\dfrac{y}{x}\right)$ into

A. $v' = \sin(v)$
B. $v' = x\sin(v)$
C. $v' + v = \sin(v)$
D. $xv' + v = \sin(v)$
E. $v' + xv = \sin(v)$

16. The solution in implicit form of

$$\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$$

is:

A. $x^2 + y^2 = x^3 + C$
B. $x^2 + y^2 = Cx^3$
C. $x^2 + x^3 = y^2 + C$
D. $Cx^2 = x^3 + y^2$
E. $x^2 + y^3 + xy^2 = C$

17. Solve the differential equation

$$(2xy + x^3)dx + (x^2 + y^3 + 2)dy = 0, \quad y(0) = 2.$$

A. $x^2y + 2y = 4$

B. $x^4 + 2y = 8$

C. $x^2y + \frac{1}{4}x^4 + \frac{1}{4}y^4 + 2y = 8$

D. $x^2y + \frac{1}{4}x^4 + \frac{1}{4}y^4 = 0$

E. $\frac{1}{4}x^4 + \frac{1}{4}y^4 = 8$

18. A ball of mass 5 kg. is thrown upward with an initial velocity of 10 (m/sec). If we neglect the air resistance, the maximum height that the ball can reach is: ($g = 9.8$ m/sec^2)

A. $\frac{100}{g}$

B. $\frac{50}{g}$

C. $50g$

D. $\frac{10}{g}$

E. $\frac{20}{g}$

19. The function $y_1 = t$ is a solution of the differential equation

$$t^2 \frac{d^2y}{dt^2} + 2t\frac{dy}{dt} - 2y = 0, \quad t > 0.$$

Find another solution $y_2(t)$ such that y_1, y_2 form a set of fundamental solutions.

A. $y_2 = t^2$

B. $y_2 = t^{-2}$

C. $y_2 = t^3$

D. $y_2 = t \ln t$

E. $y_2 = t^2 \ln t$

20. A mass weighing 2 lb stretches a spring 6 in. If the mass is pulled down an additional 3 in. and then released, and if there is no damping. Let u be the displacement from equilibrium and is measured in feet. Determine u of the mass at any time t.

A. $u(t) = \sin(8t)$

B. $u(t) = \cos(8t)$

C. $u(t) = \sin(2t) + \cos(4t)$

D. $u(t) = \frac{1}{4}\sin(\sqrt{2}t)$

E. $u(t) = \frac{1}{4}\cos(8t)$

21. Which of the following forms a fundamental set of solutions to the homogeneous differential equation $y^{(4)} - 2y'' + y = 0$.

A. $\{e^t, te^t, e^{-t}, te^{-t}\}$
B. $\{\cos t, \sin t, e^t, e^{-t}\}$
C. $\{\cos t, t\sin t, t\cos t, \sin t\}$
D. $\{e^t, e^{-t}\}$
E. $\{e^t \cos t, e^t \sin t, e^{-t} \cos t, e^{-t} \sin t\}$

22. Find the general solution of

$$y^{(4)} - 5y'' + 4y = 0.$$

A. $y(t) = c_1e^t + c_2e^{-t}$
B. $y(t) = c_1e^t + c_2e^{-t} + c_3e^{2t} + c_4e^{-2t}$
C. $y(t) = c_1e^{2t} + c_2e^{-2t}$
D. $y(t) = c_1e^t + c_2e^{2t}$
E. $y(t) = c_1e^{-t} + c_2e^{-2t}$

23. A particular solution of the equation

$$D(D+1)^2(D^2+1)y = 9\cos(t) + 2e^{-t} - 5t$$

is of the form

A. $y_p(t) = c_1 t^2\cos(t) + c_2 t^2\sin(t) + c_3 t e^{-t} + t(c_4 t + c_5)$

B. $y_p(t) = c_1 t^2\cos(t) + c_2 t^2\sin(t) + c_3 t^2 e^{-t} + c_4 t + c_5$

C. $y_p(t) = c_1 t\cos(t) + c_2 t\sin(t) + c_3 t^2 e^{-t} + c_4 t + c_5$

D. $y_p(t) = c_1 t\cos(t) + c_2 t\sin(t) + c_3 t e^{-t} + c_4 t^2 + c_5 t$

E. $y_p(t) = c_1 t\cos(t) + c_2 t\sin(t) + c_3 t^2 e^{-t} + c_4 t^2 + c_5 t$

24. Which of the following is the general solution to the system

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 \\ -4 \end{pmatrix} e^t$$

A. $c_1 \begin{pmatrix} -1 \\ 4 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t}$

B. $c_1 \begin{pmatrix} 1 \\ -4 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-2t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t$

C. $c_1 \begin{pmatrix} 1 \\ -4 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t$

D. $c_1 \begin{pmatrix} 1 \\ -4 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^t$

E. $c_1 \begin{pmatrix} 1 \\ -4 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{2t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-t}$

25. Which of the following is the general solution to the system

$$\mathbf{x}' = \begin{pmatrix} 1 & -1 \\ 5 & -3 \end{pmatrix} \mathbf{x}$$

A. $c_1 \begin{pmatrix} \cos t \\ 2\cos t + \sin t \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} \sin t \\ 2\sin t - \cos t \end{pmatrix} e^{-t}$

B. $c_1 \begin{pmatrix} 1 \\ 5 \end{pmatrix} e^{-4t} + c_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{2t}$

C. $c_1 \begin{pmatrix} \cos t \\ 2\cos t - \sin t \end{pmatrix} e^{t} + c_2 \begin{pmatrix} -\sin t \\ -2\sin t - \cos t \end{pmatrix} e^{t}$

D. $c_1 \begin{pmatrix} 1 \\ -5 \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-2t}$

E. $c_1 \begin{pmatrix} 2\cos t + \sin t \\ \cos t \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 2\sin t - \cos t \\ \sin t \end{pmatrix} e^{-t}$