MA 262, Fall 2017, Final
Version 01(Green)

INSTRUCTIONS

(1) **Switch off your phone upon entering the exam room.**
(2) **Do not open the exam booklet until you are instructed to do so.**
(3) **Before you open the booklet, fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
(4) **MARK YOUR TEST NUMBER ON THE SCANTRON**
(5) Once you are allowed to open the exam, make sure you have a complete test. There are 6 different test pages, including this cover page.
(6) Do any necessary work for each problem on the space provided or on the back of the pages of this booklet. Circle your answers in the booklet.
(7) Use a # 2 pencil to transcribe your answers to the scantron.
(8) After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

RULES REGARDING ACADEMIC DISHONESTY

(1) Do not leave the exam during the first 20 minutes of the exam.
(2) No talking. Do not seek or obtain any kind of help from anyone to answer the problems on the exam. If you need assistance, consult an instructor.
(3) Do not look at the exam of another student. You may not compare answers with other students until your exam is finished and turned in, and then only after you have left the room.
(4) Your bags must be closed throughout the exam period.
(5) Notes, books, calculators and phones must be in your bags and cannot be used.
(6) Do not handle phones or cameras or any other electric device until you have finished and turned in your exam, and then only if you have left the room.
(7) When time is called, all students must put down their writing instruments immediately. Come to the front of the room with your test and your ID to hand in.
(8) Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME ____________________

STUDENT SIGNATURE ____________________

STUDENT PUID ____________________

SECTION NUMBER ____________________

RECITATION INSTRUCTOR ____________________

(1) We consider the differential equation $x^3y''' + 3x^2y'' - 6xy' = 0$. It has three linearly independent solutions of the form x^r. The three possible values for r are

(A) $0, 3, 6$

(B) $0, \sqrt{7}, -\sqrt{7}$

(C) $-1, 3, 6$

(D) $1, 2, 6$

(E) $1, 2, 3$

(2) Identify the correct statement:

(A) If the Wronskian of $f_1(x), \ldots, f_n(x)$ is zero on an entire interval I then the functions $f_1, \ldots, f_n$ are linearly dependent on I.

(B) The matrices $A_1 = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$, $A_3 = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$ are linearly dependent.

(C) If you take fewer than n vectors in $\mathbb{R}^n$ then they will be linearly independent.

(D) If the nullspace of a matrix A is nontrivial then the columns of A are linearly dependent.

(E) The Wronskian of $f_1(x) = \sin^2(x),\ f_2(x) = \cos^2(x),\ f_3(x) = 1$ is nonzero in some point on the interval $I = [-\pi, \pi]$.

(3) According to Variation of Parameters, if $y(x) = f(x)\cos(3x) + g(x)\sin(3x)$ is a solution to

$$y'' + 9y = \frac{1}{\sin(3x)}$$

then $g(x)$ should be which one of the following options:

(A) $x/3$

(B) $\frac{1}{\sin(3x)}$

(C) $\frac{\ln|\sin(3x)|}{9}$

(D) $3/\ln(|x|)$

(E) $\frac{3}{\cos(3x)}$

(4) The appropriate trial solution to the 5-th order differential equation

$$y^{(5)} + y^{(3)} = 1 + \sin(x)$$

is

(A) $A + B\sin(x) + C\cos(s)$

(B) $Ax^3 + Bx\sin(x) + Cx\cos(x)$

(C) $A_0 + A_1x + A_2x^2 + B\sin(x) + C\cos(x)$

(D) $Ax^5 + Bx^3 + C + D\sin(x)$

(E) none of the above.

(5) Let L be the linear transformation from $\mathbb{R}^4$ to $\mathbb{R}^3$ given by $L(\vec{x}) = A\vec{x}$ where $A = \begin{bmatrix} 1 & 0 & -1 & 6 \\ 1 & 3 & -1 & 12 \\ 0 & 1 & 0 & 2 \end{bmatrix}$.

Find the correct statement:

(A) The nullspace of L is trivial.

(B) The rank of A is three.

(C) The range of L is all of $\mathbb{R}^3$.

(D) The rows of A are linearly independent.

(E) The range of L is a vector space.

(6) The determinant of $\begin{bmatrix} 3 & 3 & -4 & 1 \\ -7 & -5 & 2 & -3 \\ 5 & 4 & -3 & 2 \\ 1 & 2 & -5 & 0 \end{bmatrix}$ is

(A) 2

(B) -14

(C) 6

(D) 0

(E) 8

(7) If $A = \begin{bmatrix} 1 & -2 & -1 \\ 1 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}$ then the sum of its three eigenvalues is

(A) -4

(B) 8

(C) -2

(D) $5 + 2\sqrt{11}$

(E) -3

(8) The orthogonal trajectories to the family of curves

$$y^2 - cx^2 = 1$$

satisfy the differential equation

(A) $\dfrac{dy}{dx} = \dfrac{y^2 - 1}{xy}$

(B) $\dfrac{dy}{dx} = \dfrac{1 - y^2}{xy}$

(C) $\dfrac{dy}{dx} = \dfrac{xy}{y^2 - 1}$

(D) $\dfrac{dy}{dx} = \dfrac{xy}{1 - y^2}$

(E) $\dfrac{dy}{dx} = \dfrac{xy}{y^2 + 1}$

(9) The size of a bacterial culture grows proportionally to the number of bacteria in the culture. Initially, there were 100 bacteria, and after 4 hours, there were 150 bacteria. The doubling time of the culture, in hours, is

(A) $\frac{1}{4}\ln\left(\frac{3}{2}\right)$

(B) $4\ln\left(\frac{4}{3}\right)$

(C) $\frac{3}{2}\ln\left(\frac{1}{4}\right)$

(D) $\frac{\ln(3/2)}{\ln(4)}$

(E) $\frac{4\ln(2)}{\ln(3/2)}$

(10) The general solution to the differential equation

$$x\frac{dy}{dx} = 2x^2 + 3y$$

is

(A) $y = x^5 + cx^3$

(B) $y = cx^3 - 2x^2$

(C) $y = cx^3 + 2x^2$

(D) $y = 2x^2 + c$

(E) $y = -2x^2 + c$

(11) The general solution to the differential equation

$$\frac{dy}{dx} = \frac{x^2 + 2y^2}{xy}$$

satisfies the implicit equation

(A) $x^2 + y^2 = cx^4$

(B) $x^2 + 2y^2 = cx^6$

(C) $x^2 + y^2 = cx^2$

(D) $x^2 + 3y^2 = cx^8$

(E) $x^2 + 2y^2 = cx^4$

(12) The solution to the initial value problem

$$(2xe^y - 3)\,dx + (3y^2 + x^2e^y)\,dy = 0, \qquad y(0) = 0$$

satisfies the implicit equation

(A) $2xe^y + y^3 = 3y$

(B) $2xe^y + y^3 = 3x$

(C) $x^2e^y + y^3 = 3x$

(D) $x^2e^y + y^3 = 3y$

(E) $x^2e^y + 3xy^2 = 3$

(13) The general solution to the differential equation

$$xy'' + 2y' = 4x^2$$

is

(A) $y = \frac{1}{3}x^3 + c_1 x + c_2$

(B) $y = \frac{1}{3}x^3 + c_1 x^{-1} + c_2$

(C) $y = \frac{1}{5}x^5 + c_1 x + c_2$

(D) $y = \frac{1}{5}x^5 + c_1 x^3 + c_2$

(E) $y = 2x + c_1 x^{-1} + c_2$

(14) If

$$A = \begin{bmatrix} 1 & -2 & 0 & 4 \\ 3 & 1 & 1 & 0 \\ -1 & -5 & -1 & 8 \\ 3 & 8 & 2 & -12 \end{bmatrix}$$

then the rank of A is

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

(15) Find all values of a such that the following linear system has exactly one solution.

$$\begin{cases} ax + y - z = 2 \\ x + 2y + z = 3 \\ x - y - 2z = 3a \end{cases}$$

(A) $a = 2$

(B) $a \neq -2$

(C) $a = -2$

(D) $a \neq 2$

(E) $a = 2$ or $a = -2$

(16) If the determinant of

$$\begin{bmatrix} 2c_1 & a_1 + 3c_1 & -b_1 \\ 2c_2 & a_2 + 3c_2 & -b_2 \\ -2c_3 & -a_3 - 3c_3 & b_3 \end{bmatrix}$$

is 6, what is the determinant of

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

(A) 6

(B) 12

(C) -12

(D) -3

(E) 3

(17) Consider the vector space $V = P_2$ of all polynomials of degree at most 2 and the subset $S = \{t^2 - 2t + 1,\ t - 1,\ 3t^2 - 2t - 1\}$. Find all real numbers a such that $t^2 - at + 5 - a^2$ is in the span of S.

(A) This happens exactly when $a = 0$

(B) This happens exactly when $a = \pm 2$

(C) This happens exactly when $a = 2$ and when $a = -1$

(D) This happens exactly when $a = 2$ and when $a = -3$

(E) This happens for every real number a

(18) Assume that the 4×4 matrix A is row equivalent to the matrix B, where

$$B = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Which of the following statements is true?

(A) B is not the reduced row echelon form of A.

(B) $\det(A) \neq 0$.

(C) The null space of A has dimension 3.

(D) The column space of A has dimension 2.

(E) If $A\vec{x} = \vec{b}$ is consistent for some $\vec{b} \neq \vec{0}$, then it has infinitely many solutions.

(19) Let $m > n$ be two positive integers. Which of the following statements are always true?

(I) Any linear system of n equations in m variables is consistent.

(II) Any linear system of m equations in n variables is consistent.

(III) Any homogeneous linear system of n equations in m variables has infinitely many solutions.

(IV) Any homogeneous linear system of m equations in n variables has infinitely many solutions.

(A) (I) and (III), but not (II) or (IV)

(B) (III) only

(C) (III) and (IV), but not (I) or (II)

(D) (II), (III) and (IV), but not (I)

(E) (I), (II), (III) and (IV)

(20) Consider the differential equation

$$x^2(1+x)y'' - 2xy' + 2y = 0, \quad x > 0.$$

It turns out that the function $y_1(x) = x$ is a solution to this equation.

If $y_2 = xu(x)$ is also a solution, then $u(x)$ must satisfy the differential equation

(A) $u'' + (x+1)^{-2}u' = 0$

(B) $u'' + 2x^{-2}u' = 0$

(C) $u'' + (x^2+1)^{-1}u' = 0$

(D) $6u'' + 2xu' = 0$

(E) $u'' + 2(x+1)^{-1}u' = 0$

(21) The free oscillation of a spring-mass system with negligible damping is determined by

$$\frac{d^2x}{dt^2} + \beta \frac{dx}{dt} = -\kappa x + F(t)$$

with

(A) $F(t) \neq 0$, $\kappa \neq 0$, and $\beta = 0$

(B) $F(t) \neq 0$, $\kappa = 0$, and $\beta \neq 0$

(C) $F(t) = 0$, $\kappa \neq 0$, and $\beta \neq 0$

(D) $F(t) = 0$, $\kappa \neq 0$, and $\beta = 0$

(E) $F(t) \neq 0$, $\kappa \neq 0$, and $\beta \neq 0$

(22) If

$$y_p(x) = A\ \cos(x) + B\ \sin(x)$$

is a particular solution of the differential equation

$$y'' - 3y' + 2y = 10\ \cos(x),$$

then

(A) $A = 1, \quad B = -3$

(B) $A = 2, \quad B = 3$

(C) $A = -1, \quad B = -3$

(D) $A = -3, \quad B = 1$

(E) $A = 3, \quad B = -1$

(23) The system $\mathbf{x}' = A\mathbf{x} + \begin{bmatrix} 5e^t \\ 7e^t \end{bmatrix}$ has a fundamental matrix

$$X(t) = \begin{bmatrix} e^t & 2e^{2t} \\ e^t & 3e^{2t} \end{bmatrix}.$$

We use the method of variation of parameters in order to find a particular solution $x_p(t)$ of the form $\mathbf{x}_p(t) = X(t)\begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$. Then $u_2(t) - u_1(t)$ equals

(A) $-2e^{-t} - t$

(B) $e^{-t} - 2$

(C) $-e^{-t} + t$

(D) $2e^{-t} - t$

(E) $e^{-2t} + 1$

(24) Given is the differential equation

$$y'' + ay' + by = F(t),$$

where a, b are some constants. We set $x_1 = y$, $x_2 = \dfrac{dy}{dt} = y'$ and $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. The differential equation is equivalent to

$$X' = AX + \begin{bmatrix} 0 \\ F(t) \end{bmatrix},$$

where A is a 2×2 matrix. Then $\det(A)$ is

(A) b

(B) a

(C) ab

(D) $-ab$

(E) 0

(25) If $\mathbf{x}_1(t) = \begin{bmatrix} t+1 \\ t-1 \\ 2t \end{bmatrix}$, $\mathbf{x}_2(t) = \begin{bmatrix} e^t \\ e^{2t} \\ e^{3t} \end{bmatrix}$, $\mathbf{x}_3(t) = \begin{bmatrix} 1 \\ \sin t \\ \cos t \end{bmatrix}$, and $W[\mathbf{x}_1, \ldots, \mathbf{x}_n](t)$ is the Wronskian of these vector functions, then $W[\mathbf{x}_1, \ldots, \mathbf{x}_n](0)$ is

(A) 1

(B) -1

(C) 0

(D) 3

(E) not defined