MA 262 Fall 2002
FINAL EXAM INSTRUCTIONS

NAME ______________________ INSTRUCTOR ______________________

1. You must use a #2 pencil on the mark–sense sheet (answer sheet).

2. On the mark–sense sheet, fill in the instructor's name and the course number.

3. Fill in your name and student identification number and blacken in the appropriate spaces.

4. Mark in the section number, the division and section number of your class. For example, for division 02, section 03, fill in 0203 and blacken the corresponding circles, including the circles for the zeros. (If you do not know your division and section number ask your instructor.)

5. Sign the mark–sense sheet.

6. Fill in your name and your instructor's name above and on the first page of the question sheets.

7. There are 25 questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25. Do all your work on the question sheets. Turn in both the mark–sense sheets and the question sheets when you are finished.

8. No partial credit will be given, but if you show your work on the question sheets it may be considered if your grade is on the borderline.

9. NO CALCULATORS, BOOKS OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

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1. If $y' = \dfrac{3(x+1)^2}{y}$, $y(-1) = 2$, then $y(0) =$

A. 3

B. $\sqrt{3}$

C. $\sqrt{6}$

D. 8

E. $\sqrt{10}$

2. If $xy' - 3y = x^3$ and $y(1) = 1$ then $y(e) =$

A. e^4

B. e^{-2}

C. $2e^{-3}$

D. $2e^3$

E. e^3

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3. The general solution of

$$\frac{dy}{dx} = \frac{2xy + 3x^2}{y^2 - x^2}$$

is

A. $\frac{y^3}{3} - x^2y - x^3 = c$

B. $\frac{y^3}{3} + x^2y + x^3 = c$

C. $2xy + y^2 + 3x^2 = c$

D. $\log(x^2 + y^2) = c$

E. none of the above

4. The population of a certain city is increasing at a rate **inversely proportional** to the population. At $t = 0$ the population is 1000 and at $t = 1$ it is 2000. The population as a function of t is $p(t) =$

A. $1000\sqrt{3t+1}$

B. $\dfrac{1000}{\sqrt{1 - \frac{3}{4}t}}$

C. $1000(1+t)$

D. $\dfrac{1000}{1 - \frac{1}{4}t}$

E. $1000(1+t^2)$

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5. A rocket traveling straight up has velocity v_0 when it is at a distance of $2R$ from the surface of the earth where R is the radius of the earth. The engines are shut off and the only force on the rocket is due to the gravitational attraction of the earth. The velocity v of the rocket when it is a distance x from the surface of the earth satisfies a differential equation

$$v\frac{dv}{dx} = \frac{-gR^2}{(x+R)^2},$$

where g is a constant. For what values of v_0 will the velocity be positive for all x?

A. $v_0 > \sqrt{2gR}$

B. $v_0 > \sqrt{gR}$

C. $v_0 > \sqrt{\frac{2gR}{3}}$

D. $v_0 > \sqrt{\frac{gR}{2}}$

E. $v_0 > \sqrt{\frac{gR}{3}}$

6. Initially a 100-gallon tank is half full of pure water. A salt solution containing 0.2 lb. of salt per gallon runs into the tank at a rate of 3 gallons per minute. The well mixed solutions runs out of the tank at a rate of 2 gallons per minute. Let $x(t)$ be the amount of salt in the tank at time t. Then $x(t)$ satisfies the differential equation

A. $\frac{dx}{dt} = 0.4 - \frac{3x}{50+t}$

B. $\frac{dx}{dt} = 0.6 - \frac{2x}{50-t}$

C. $\frac{dx}{dt} = 0.6 + \frac{2x}{50+t}$

D. $\frac{dx}{dt} = 0.6 - \frac{2x}{50+t}$

E. $\frac{dx}{dt} = 0.4 - \frac{2x}{50+t}$

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7. If $A = \begin{bmatrix} 3 & 4 & 7 \\ 2 & 6 & 1 \\ 3 & 14 & -1 \end{bmatrix}$, then $\det(-2A) =$

A. 60

B. -60

C. -240

D. 240

E. 30

8. If A and B are 3×3 matrices such that $\det A = 3$, $\det B = -4$, then $\det(-2A^{-1}B) =$

A. $-\frac{32}{3}$

B. $\frac{32}{3}$

C. 96

D. $-\frac{8}{3}$

E. $\frac{8}{3}$

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9. If $A = \begin{bmatrix} e^t & e^{-2t} \\ e^t & -2e^{-2t} \end{bmatrix}$, then det $(A^{-1}) =$

A. $-\frac{1}{3}e^t$

B. $-\frac{1}{3}e^{-t}$

C. $-3e^t$

D. $3e^t$

E. $-3e^{-t}$

10. Determine all values of k so that the system

$$\begin{aligned} x_1 + \ x_3 &= 0 \\ x_1 + 2x_2 + kx_3 &= 0 \\ kx_1 + kx_2 + 6x_3 &= k + 4 \end{aligned}$$

has infinitely many solutions.

A. $k \neq -4$

B. $k \neq 3$ and $k \neq -4$

C. $k = 3$ and $k = -4$

D. $k = 3$

E. $k = -4$

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11. Which of the following sets is a subspace of the given vector space?

A. $V = C^3(\mathbb{R})$, $S = \{y''' - (x^2 - 1)y'' + 3x(y' - 2) - y = 0\}$

B. $V = C^3(\mathbb{R})$, $S = \{y''' - (x^2 - 1)y'' + 3xy' - y = 0\}$

C. $V = \mathbb{R}^3$, $S = \{(x, y, z) \in \mathbb{R}^3 : 3x - y = z + 1\}$

D. $V = M_{2\times 2}(\mathbb{R})$, $S = \left\{A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{2\times 2}(\mathbb{R}) : a + d = 1\right\}$

E. $V = C^1([-1, 1])$, $S = \{f \in V, f'(-1) = 2f'(1) - 1\}$

12. Consider the system $A\mathbf{x}=\mathbf{b}$ given by

$$\begin{bmatrix} 3 & 0 & -2 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}.$$

then $\mathbf{x}^T = [x_1, x_2, x_3]$ is

A. $[1, 2, 2]$

B. $[-7, -24, -10]$

C. $[1, -2, 2]$

D. $[-1, -4, 1]$

E. $[3, 0, 5]$

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13. The vectors $(1,2,1)$, $(3,4,5)$, and $(2,-2,k)$ are linearly **dependent** if k equals

A. 4

B. 8

C. -5

D. -1

E. 0

14. Let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ -3 & 3 & -6 \end{bmatrix},$$

and let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation given by $T(x) = Ax$. Then $\text{Ker}(T)$ and $\text{Rng}(T)$ are

A. $\text{Ker}(T) = \text{span}\{(0,0,1),(1,0,-2)\}$, $\text{Rng}(T) = \text{span}\{(0,0,-1),\ (1,2,3)\}$

B. $\text{Ker}(T) = \text{span}\{(1,1,0),(-2,0,-1)\}$, $\text{Rng}(T) = \text{span}\{(0,-3),\ (1,2)\}$

C. $\text{Ker}(T) = \text{span}\{(1,1,0),(-2,0,1)\}$, $\text{Rng}(T) = \text{span}\{(1,-3)\}$

D. $\text{Ker}(T) = \text{span}\{(1,0),(0,-1)\}$, $\text{Rng}(T) = \text{span}\{(1,1,0),(2,0,2)\}$

E. $\text{Ker}(T) = \text{span}\{(0,-3,0),(-2,0,0)\}$, $\text{Rng}(T) = \text{span}\{(0,0)\}$

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15. A basis for the kernel of the linear transformation $T : C^2(\mathbb{R}) \to C(\mathbb{R})$ given by $T(y) = y'' + 4y' + 8y$ is

A. $\{e^{2t}, te^{2t}\}$

B. $\{e^{-2t}\cos t,\ e^{-2t}\sin t\}$

C. $\{e^{-4t}\cos 2t,\ e^{-4t}\sin 2t\}$

D. $\{e^{-4t}\cos t,\ e^{-4t}\sin t\}$

E. $\{e^{-2t}\cos 2t,\ e^{-2t}\sin 2t\}$

16. The general solution of $y'' + ay' + by = 0$ is $y = c_1e^x + c_2e^{2x}$. To find a particular solution by the method of undetermined coefficients of the equation

$$y'' + ay' + by = e^x + e^{3x} + 1,$$

one should try a solution of the form

A. $c_1e^x + c_2e^{3x} + c_3$

B. $c_1xe^x + c_2e^{3x} + c_3$

C. $c_1xe^x + c_2e^{3x} + c_3x$

D. $c_1xe^x + c_2e^{3x}$

E. $c_1e^x + c_2e^{3x} + c_3x + c_4x^2$

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17. A trial solution to use for finding a particular solution of the differential equation $(D^2 - 1)(D^2 - 4D + 3)y = \cos x - xe^x$ is:

A. $C_1 \cos x + C_2 \sin x + C_3 x^2 e^x + C_4 x^3 e^x$

B. $C_1 \cos x + C_2 \sin x + C_3 x e^x + C_4 x^2 e^x$

C. $C_1 x \cos x + C_2 x \sin x + C_3 x e^x + C_4 x^2 e^x + C_5 x^3 e^x$

D. $C_1 \cos x + C_2 \sin x + C_3 x e^x + C_4 e^{3x}$

E. $C_1 \cos x + C_2 \sin x + C_3 e^x + C_4 x e^x + C_5 x^2 e^x$

18. If $y = u_1 y_1 + u_2 y_2$, where $y_1 = e^t$ and $y_2 = t$, is a particular solution of the equation

$$y'' + \frac{t}{1-t} y' - \frac{1}{1-t} y = 2(1-t)e^{-t}, \quad 0 < t < 1,$$

then by applying the method of variation of parameters one finds that $u_2 =$

A. $-2e^{-t}$

B. $2e^{-t}$

C. e^{-t}

D. $\frac{1}{2}e^{-t}$

E. te^{-t}

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19. One solution of the differential equation

$$x^3 y'' + xy' - y = 0, \quad x > 0$$

is $y_1 = x$. Another solution is of the form $y_2 = vx$ where v satisfies the differential equation

A. $v'' + xv' = 0$

B. $x^2 v'' + 2xv' = 0$

C. $x^2 v'' + (x+1)v' = 0$

D. $x^2 v'' + (2x+1)v' = 0$

E. $(2x+1)v'' + x^2 v' = 0$

20. Let $y(x)$ be the solution to the initial value problem

$$y'' - 3y' + 2y = 4x, \qquad y(0) = 4, \; y'(0) = 3.$$

What is $y(1)$?

A. $e^2 + 3$

B. $e + e^2 + 1$

C. $e^2 + 1$

D. $e + e^2$

E. $e + 5$

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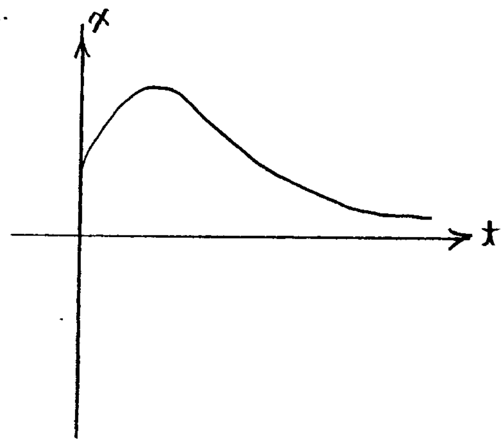
21. The oscillation of a spring-mass system is determined by $\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = 0$, with initial conditions $x(0) = 1$ and $\frac{dx}{dt}(0) = -3$. Then a sketch of the motion $x(t)$ is

A.



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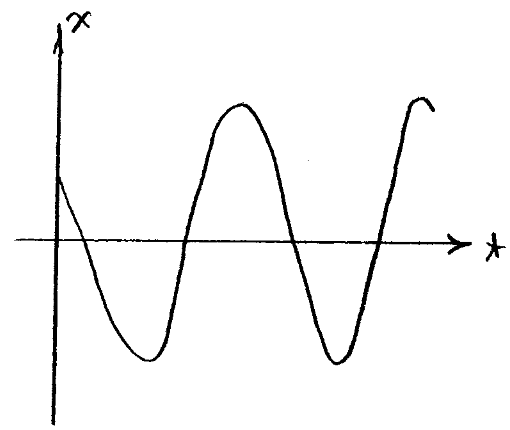

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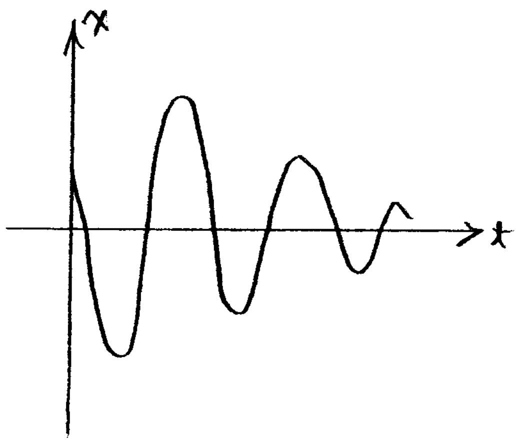

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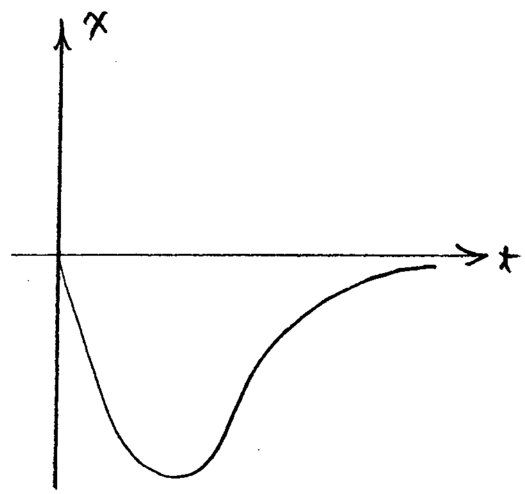
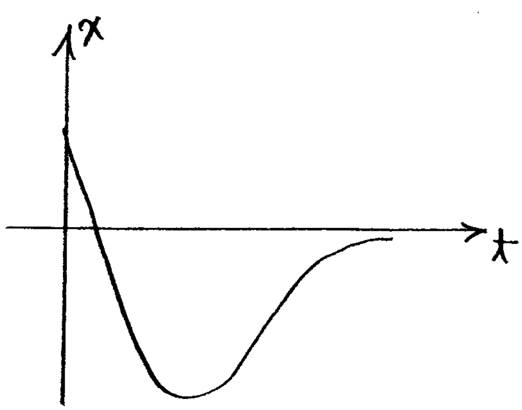


E.



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22. It is observed that an eigenvalue of the matrix

$$\begin{bmatrix} -1 & 0 & 0 \\ 2 & 1 & 4 \\ 0 & -1 & -3 \end{bmatrix}$$

is $\lambda = -1$. Let m_{-1} denote the multiplicity of this eigenvalue, and d_{-1} denote the dimension of the eigenspace corresponding to this eigenvalue, then which one of the following is true?

A. $m_{-1} = 3,\ d_{-1} = 1$

B. $m_{-1} = 3,\ d_{-1} = 2$

C. $m_{-1} = 3,\ d_{-1} = 3$

D. $m_{-1} = 2,\ d_{-1} = 1$

E. $m_{-1} = 2,\ d_{-1} = 2$

23. Determine all values of k so that the matrix $\begin{bmatrix} 0 & 2 & k \\ 0 & 2 & k \\ 0 & 2 & k \end{bmatrix}$ is **defective**.

A. $k = 0$

B. $k = 2$

C. $k = -2$

D. no k

E. all k's

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24. A fundamental matrix of the following first order system of differential equations

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 2 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

is

A. $\begin{bmatrix} 0 & -e^{2t} & e^{4t} \\ e^{2t} & 0 & 0 \\ 0 & e^{2t} & e^{4t} \end{bmatrix}$

B. $\begin{bmatrix} 0 & -e^{-2t} & e^{4t} \\ e^{-2t} & 0 & 0 \\ 0 & e^{-2t} & e^{4t} \end{bmatrix}$

C. $\begin{bmatrix} 3e^{2t} & e^{-2t} & -e^{4t} \\ -4e^{2t} & 0 & 0 \\ e^{2t} & e^{-2t} & e^{4t} \end{bmatrix}$

D. $\begin{bmatrix} 3e^{2t} & -e^{-2t} & e^{4t} \\ -4e^{2t} & 0 & 0 \\ e^{2t} & e^{-2t} & e^{4t} \end{bmatrix}$

E. $\begin{bmatrix} 3e^{-2t} & -e^{-2t} & e^{4t} \\ -4e^{-2t} & 0 & 0 \\ e^{-2t} & e^{-2t} & e^{4t} \end{bmatrix}$

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25. If a fundamental matrix for $\mathbf{x}' = A\mathbf{x}$ is $X(t) = \begin{bmatrix} -e^{-2t} & 0 \\ e^{-2t} & e^{2t} \end{bmatrix}$, then the general solution to the system of differential equations $\mathbf{x}' = A\mathbf{x} + \begin{bmatrix} 0 \\ e^t \end{bmatrix}$ is

A. $\begin{bmatrix} -e^{-2t} & 0 \\ e^{-2t} & e^{2t} \end{bmatrix} \left\{ \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} 0 \\ e^{-t} \end{bmatrix} \right\}$

B. $\begin{bmatrix} -e^{-2t} & 0 \\ e^{-2t} & e^{2t} \end{bmatrix} \left\{ \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} - \begin{bmatrix} 0 \\ e^{t} \end{bmatrix} \right\}$

C. $\begin{bmatrix} -e^{2t} & 0 \\ e^{-2t} & e^{-2t} \end{bmatrix} \left\{ \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} 0 \\ e^{-t} \end{bmatrix} \right\}$

D. $\begin{bmatrix} -e^{2t} & 0 \\ e^{-2t} & e^{-2t} \end{bmatrix} \left\{ \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} - \begin{bmatrix} 0 \\ e^{-t} \end{bmatrix} \right\}$

E. $\begin{bmatrix} -e^{-2t} & 0 \\ e^{-2t} & e^{2t} \end{bmatrix} \left\{ \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} - \begin{bmatrix} 0 \\ e^{-t} \end{bmatrix} \right\}$