MA 262 Fall 2001
FINAL EXAM INSTRUCTIONS

NAME ______________________ INSTRUCTOR ______________________

INSTRUCTIONS:

1. You must use a #2 pencil on the mark–sense sheet (answer sheet).

2. On the mark–sense sheet, fill in the instructor's name and the course number.

3. Fill in your name and student identification number and blacken in the appropriate spaces.

4. Mark in the section number, the division and section number of your class. For example, for division 02, section 03, fill in 0203 and blacken the corresponding circles, including the circles for the zeros. (If you do not know your division and section number ask your instructor.)

5. Sign the mark–sense sheet.

6. Fill in your name and your instructor's name above.

7. There are 25 questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25. Do all your work on the question sheets. Turn in both the mark–sense sheets and the question sheets when you are finished.

8. No partial credit will be given, but if you show your work on the question sheets it may be considered if your grade is on the borderline.

9. NO CALCULATORS, BOOKS OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

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1. The general solution of $y' - \dfrac{2y}{1+x} = (1+x)^2$ is

A. $y(x) = \frac{1}{5}(1+x)^3 + \dfrac{c}{(1+x)^2}$

B. $y(x) = \frac{1}{5}(1+x)^3 + c$

C. $y(x) = \frac{1}{3}(1+x)^3 + \dfrac{c}{(1+x)^2}$

D. $y(x) = (x+c)(1+x)^2$

E. $y(x) = x(1+x)^2 + \dfrac{c}{(1+x)^2}$

2. Initially a tank holds 100 gallons of pure water. A salt solution containing $\frac{1}{5}$ lb. of salt per gallon runs into the tank at a rate of 3 gallons per minute. The well mixed solution runs out of the tank at a rate of 2 gallons per minute. Let $x(t)$ be the amount of salt in the tank at time t. Find a differential equation satisfied by $x(t)$.

A. $\dfrac{dx}{dt} = \dfrac{1}{5} - \dfrac{3x}{100+t}$

B. $\dfrac{dx}{dt} = \dfrac{2}{5} - \dfrac{3x}{100+t}$

C. $\dfrac{dx}{dt} = \dfrac{3}{5} - \dfrac{2x}{100}$

D. $\dfrac{dx}{dt} = \dfrac{1}{5} - \dfrac{2x}{100+t}$

E. $\dfrac{dx}{dt} = \dfrac{3}{5} - \dfrac{2x}{100+t}$

3. The general solution of $(2xy + \cos y)dx + (x^2 - x\sin y - 2)dy = 0$ is

A. $x^2y + x\cos y - 2y = c$

B. $x^2y + \cos y - 2y = c$

C. $x^2y + x\sin y - 2y = c$

D. $x^2y - x\cos y - 2y = c$

E. $x^2y^2 + x\cos y - 2y = c$

4. The solution to the initial value problem

$$y'' - 3y' + 2y = 0 \qquad y(0) = 0 \qquad y'(0) = 1$$

has value $y(\ln 2) =$

A. 0

B. 2

C. 4

D. 6

E. 8

5. The constants a, b, c are real and the roots of the equation $ar^2 + br + c = 0$ are $r = 1 \pm 2i$. The general solution of the differential equation $ay'' + by' + cy = 0$ is $y =$

A. $e^x(c_1 \cos 2x + c_2 \sin 2x)$

B. $e^{2x}(c_1 \sin x + c_2 \cos x)$

C. $c_1 e^x + c_2 e^{2x}$

D. $c_1 \cos x + c_2 \sin x$

E. $c_1 \cos 2x + c_2 \sin 2x$

6. If y_1 and y_2 are two solutions of the nonhomogeneous differential equation

$$y''' + a_1(x)y' + a_2(x)y = F(x)$$

then which of the following functions is also a solution of the same differential equation.

A. $y_1 - y_2$

B. $y_1 + y_2$

C. $\frac{1}{2}(y_1 + y_2)$

D. $\frac{1}{2}(y_1 - y_2)$

E. $y_1 - 2y_2$

7. If $y = u_1 y_1 + u_2 y_2$ where $y_1 = e^{2x}$ and $y_2 = e^{-2x}$ is a particular solution of

$$y'' - 4y = 4\tan x$$

then u_1 and u_2 are determined by

A. $u_1' = e^{-2x}\tan x \quad u_2' = -e^{2x}\tan x$

B. $u' = -e^{-2x}\sec^2 x \quad u_2' = e^{2x}\sec^2 x$

C. $u_1' = -2\sin 2x \tan x \quad u_2' = 2\cos 2x \tan x$

D. $u_1' = 2\sin 2x \tan x \quad u_2' = -2\cos 2x \tan x$

E. $u_1' = \tan x \quad u_2' = 0$

8. The function $y_1 = e^x$ is a solution of

$$xy'' - 2y' + (2 - x)y = 0$$

If $y_2 = u(x)e^x$ is the other linearly independent solution, then u must satisfy

A. $xu'' + u' = 0$

B. $xu'' + 2(x-1)u' = 0$

C. $xu'' + (x-2)u' = 0$

D. $xu'' + 2(x-2)u' = 0$

E. $xu'' + xu' + u = 0$

9. The general solution of $y'' - 3y' - 4y = 0$ is $y = c_1 e^{-x} + c_2 e^{4x}$. Which form would you use to determine a particular solution to the equation $y'' - 3y' - 4y = xe^{-x} + \cos 2x$ by undetermined coefficients?

A. $Axe^{-x} + B\cos 2x$

B. $Axe^{-x} + B\cos 2x + C\sin 2x$

C. $x(A + Bx)e^{-x} + C\cos 2x$

D. $x(A+Bx)e^{-x} + C\cos 2x + D\sin 2x$

E. $Ax^2e^{-x} + B\cos 2x + C\sin 2x$

10. Find all values of a such that the following system of equations has exactly one solution.

$$\begin{cases} x + y - z = 2 \\ x + 2y + z = 3 \\ x + y + (a^2 - 5)z = a \end{cases}$$

A. $a \neq 2$ and $a \neq -2$

B. $a = 2$ or $a = -2$

C. $a = -2$

D. $a = \pm\sqrt{5}$

E. $a \neq 0$

11. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$ and let $B = A^{-1}$. Then the entry b_{13} of A^{-1} is

A. 0

B. 1

C. -1

D. 2

E. -2

12. The Wronskian of the functions $\{x, \sin x, \cos x\}$ is equal to

A. $x(\cos x)(\sin x)$

B. $x(\cos^2 x - \sin^2 x)$

C. 0

D. x

E. $-x$

13. Let S be the subspace of $\mathbb{R}^3$ consisting of all vectors $\mathbf{x}$ of the form $\mathbf{x} = (r + s,\ r - s,\ 2r + 2s)$, r, s real. A basis for S is the pair

A. $(1, 1, 2),\ (1, -1, 2)$

B. $(2, 0, 4),\ (2, 0, 2)$

C. $(1, 1, 2),\ (-1, 1, 2)$

D. $(2, 1, 1),\ (2, -1, 1)$

E. $(1, 1, 2),\ (2, -1, 1)$

14. Which of the following are subspaces of the set of all continuous function on $(-\infty, \infty)$

(i) functions f such that $f(x) = f(-x)$

(ii) functions f such that $f'' + 2f' + x^2 f = x^2$

(iii) functions f such that $f(1) = 0$

A. (i) and (ii)

B. (i) and (iii)

C. (ii) and (iii)

D. (iii) only

E. (i) only

15. Let S denote the set of all polynomials of degree less than or equal to 4. What is the dimension of S?

A. 1

B. 2

C. 3

D. 4

E. 5

16. Let L denote the differential operator $(D-1)^2(D^2+1)$. Then a basis for $\ker(L)$ is:

A. $e^{x-1}, e^{(x-1)^2}, \sin x, \cos x$

B. $e^{x^2}, \sin x, \cos x$

C. $e^x, xe^x, \sin x, \cos x$

D. e^x, e^{-x}, e^{x^2+1}

E. $e^{(x-1)^2}, e^{x^2+1}$

17. Determine all values of k such that the vectors $(1,-1,0)$, $(1,2,2)$, $(0,3,k)$ are a basis for $\mathbb{R}^3$.

A. $k=1$

B. $k=2$

C. $k \neq 2$

D. $k \neq 1$

E. $k \neq 3$

18. Let $T : \mathbb{R}^5 \to \mathbb{R}^2$ be the linear transformation given by $T(x) = Ax$ where $A = \begin{bmatrix} 1 & 2 & -3 & 4 & 5 \\ 2 & -7 & 11 & 13 & 19 \end{bmatrix}$. Find the dimension of the kernel of T.

A. 0

B. 1

C. 2

D. 3

E. 4

19. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation defined by $T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\2\\3\end{bmatrix}$ and $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}0\\3\\1\end{bmatrix}$. What is $T\left(\begin{bmatrix}1\\2\end{bmatrix}\right)$?

A. $\begin{bmatrix}4\\6\\3\end{bmatrix}$

B. $\begin{bmatrix}1\\0\\1\end{bmatrix}$

C. $\begin{bmatrix}2\\2\\8\end{bmatrix}$

D. $\begin{bmatrix}1\\5\\4\end{bmatrix}$

E. $\begin{bmatrix}2\\1\\5\end{bmatrix}$

20. The product of the eigenvalues of the matrix $M = \begin{bmatrix}1 & 1\\-1 & 4\end{bmatrix}$ is:

A. 2

B. 3

C. 4

D. 5

E. 6

21. The matrix $A = \begin{bmatrix}1 & -2\\5 & -5\end{bmatrix}$ has eigenvalue $-2+i$. Which of the following is an eigenvector of A?

A. $(5, 5-i)$

B. $(5, 3-i)$

C. $(3+i, 5)$

D. $(5-i, 3)$

E. $(3+i, 3-i)$

22. The eigenvalues of the matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ -2 & 3 & -2 \\ 2 & 0 & 5 \end{bmatrix}$ are 3 and 5. One of the associated eigenspaces has dimension one. This eigenspace has basis:

A. $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

B. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

E. $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

23. A 2×2 matrix A has eigenvalues -1, 4, with corresponding eigenvectors $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$. The general solution to $\mathbf{x}' = A\mathbf{x}$ is

A. $\begin{cases} x_1 = 3c_1e^{-t} + c_2e^{4t} \\ x_2 = 2c_1e^{-t} - c_2e^{4t} \end{cases}$

B. $\begin{cases} x_1 = 2c_1e^{-t} + 3c_2e^{4t} \\ x_2 = c_1e^{-t} - 2c_2e^{4t} \end{cases}$

C. $\begin{cases} x_1 = 3c_1 \cos t + c_2 \sin t \\ x_2 = 2c_1 \cos t - c_2 \sin t \end{cases}$

D. $\begin{cases} x_1 = c_1e^{3t} - c_2e^{4t} \\ x_2 = c_1e^{2t} + c_2e^{-t} \end{cases}$

E. $\begin{cases} x_1 = 3c_1e^{-t} + c_2e^{4t} \\ x_2 = 2c_1e^{-t} + c_2e^{4t} \end{cases}$

24. The real 2×2 matrix A has eigenvalues $1+i$ and $1-i$, with corresponding eigenvectors $\begin{bmatrix} i \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -i \\ 1 \end{bmatrix}$. If $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ is the general solution $\mathbf{x}' = A\mathbf{x}$, and $x_2(t) = e^t(c_1 \cos t + c_2 \sin t)$, then $x_1(t)$ is

A. $e^{-t}(c_2 \cos t - c_1 \sin t)$

B. $e^{-2t}(c_1 \sin t - c_2 \cos t)$

C. $e^t(c_1 \sin t - c_2 \cos t)$

D. $e^t(c_1 \sin t - 2c_2 \cos t)$

E. $e^t(c_2 \cos t - c_1 \sin t)$

25. If a fundamental matrix for the system $\mathbf{x}' = A\mathbf{x}$ is $\mathbf{X}(t) = \begin{bmatrix} e^t & 0 \\ 0 & e^{-t} \end{bmatrix}$, find a particular solution, $\mathbf{y}_p(t)$ of $\mathbf{x}' = A\mathbf{x} + \begin{bmatrix} e^t \\ 2 \end{bmatrix}$, such that $\mathbf{y}_p(0) = \mathbf{0}$.

A. $\begin{bmatrix} t \\ 2e^t - 2 \end{bmatrix}$

B. $\begin{bmatrix} 2e^t \\ t \end{bmatrix}$

C. $\begin{bmatrix} 2 - 2e^t \\ 2 - 2e^{-t} \end{bmatrix}$

D. $\begin{bmatrix} te^t \\ 2 - 2e^{-t} \end{bmatrix}$

E. $\begin{bmatrix} te^t \\ 2 \end{bmatrix}$