**MA262 — EXAM II — SPRING 2022 — APRIL 4, 2022**
**TEST NUMBER 01**

**INSTRUCTIONS:**

1. **Do not open the exam booklet until you are instructed to do so.**
2. **Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
3. **MARK YOUR TEST NUMBER ON YOUR SCANTRON**
4. Once you are allowed to open the exam, make sure you have a complete test. There are 8 different test pages including this cover page.
5. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. No extra paper is allowed. Circle your answers on this test booklet.
6. There are eleven problems, each problem is worth 9 points and everyone gets one point. The maximum possible score is 100 points. No partial credit.
7. After you finish the exam, hand in your scantron and your test booklet to one of the proctors.

**RULES REGARDING ACADEMIC DISHONESTY:**

1. Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only your instructor.
2. Do not look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. Do not consult notes, books, calculators.
4. Do not handle phones or cameras, or any electronic device until after you have finished your exam, handed it in to your instructor and left the room.
5. **After time is called, the students have to put down all writing instruments.**
6. **At the end of the exam period, students SHOULD NOT stand in lines to hand in their exams. They should remain in their seats, while the proctors will collect the scantrons and the exams.**
7. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: ______________________

STUDENT SIGNATURE: ______________________

STUDENT ID NUMBER: ____________ SECTION NUMBER ____________

RECITATION INSTRUCTOR: ______________________

**1.** Which of the following statements are correct?

I. If $\mathbf{A}$ is a $m \times n$ matrix and $\mathbf{b}$ a vector in $\mathbb{R}^m$ which is not equal to zero, the set of vectors $\mathbf{x}$ in $\mathbb{R}^n$ that satisfy the system $\mathbf{Ax} = \mathbf{b}$ is a subspace of $\mathbb{R}^n$.

II. The set of vectors $(x, y)$ in $\mathbb{R}^2$ satisfying $x^2 - y^2 = 0$ is a subspace of $\mathbb{R}^2$.

III. If $\mathbb{M}_2(\mathbb{R})$ is the space of all $2 \times 2$ matrices with real entries, then the set of all matrices in $\mathbb{M}_2(\mathbb{R})$ which satisfy $\mathbf{A} = -\mathbf{A}^T$ is a subspace of $\mathbb{M}_2(\mathbb{R})$.

IV. If $\mathbb{M}_3(\mathbb{R})$ is the space of all $3 \times 3$ matrices with real entries, then the set of all matrices $A$ in $\mathbb{M}_3(\mathbb{R})$ such that $\det(\mathbf{A}) = 1$ is a subspace of $\mathbb{M}_3(\mathbb{R})$.

V. If $\mathbb{P}_2(\mathbb{R})$ is the space of polynomials of degree less than or equal to 2 with real coefficients, then the set of polynomials in $\mathbb{P}_2(\mathbb{R})$ which are of the form $ax^2 + b$ with $a$ and $b$ real, form a subspace of $\mathbb{P}_2(\mathbb{R})$.

A. Only I, III and V

B. Only II and III

C. Only III and V

D. Only I and IV

E. Only I, IV and V

**2.** Find a basis for the null space of the matrix $A$ below.

$$A = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 1 & 3 & 4 & 2 \\ 2 & 4 & 6 & -1 \end{bmatrix}$$

A. $\{(1, 2, 1, 1)\}$

B. $\{(1, 1, -1, 0)\}$

C. $\{(1, 1, -1, 0), (1, 2, 1, 1)\}$

D. $\{(1, -1, -1, 0), (1, 2, 1, 1)\}$

E. $\{(1, -1, -1, 0), (1, 2, 1, 1), (1, 0, 0, 1)\}$

**3.** Let

$$\mathbf{V}_1 = (-1, 0, 0, 1), \quad \mathbf{V}_2 = (1, 3, -1, 0), \quad \mathbf{V}_3 = (1, 0, 1, 2) \text{ and } \mathbf{V}_4 = (0, -1, 2, 3).$$

Determine the dimension of $\text{Span}(\{\mathbf{V}_1, \mathbf{V}_2, \mathbf{V}_3, \mathbf{V}_4\})$ in $\mathbb{R}^4$.

A. 1

B. 2

C. 3

D. 4

E. 5

**4.** Let $\mathbf{A}$ be a $6 \times 4$ matrix. We assume that $\text{Nullity}(\mathbf{A}) = 0$ (the dimension of the null space of $\mathbf{A}$ is equal to zero. Then the following can be said about $\text{Colspace}(\mathbf{A})$ (the space spanned by the columns of $\mathbf{A}$)?

A. It is a subspace of $\mathbb{R}^6$ of dimension 4

B. It is a subspace of $\mathbb{R}^4$ of dimension 2

C. It is a subspace of $\mathbb{R}^6$ of dimension 3

D. It is a subspace of $\mathbb{R}^4$ of dimension 3

E. It is a subspace of $\mathbb{R}^6$ of dimension 5

**5.** Suppose that

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & r \end{bmatrix} = 5.$$

Find

$$\det \begin{bmatrix} 2g + 3a & 2h + 3b & 2r + 3c \\ d & e & f \\ 2a & 2b & 2c \end{bmatrix}.$$

A. $-20$

B. $-10$

C. 5

D. 10

E. 20

**6.** Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$ and $C = B^{-1}A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then $a+b+c+d =$ (the sum of all the elements of the matrix $C$ is equal to).

A. 2

B. $-1$

C. $-4$

D. 0

E. $-2$

**7.** Let

$$A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

and let $A^{-1} = (a_{jk})$. Given that $\det(A) = 6$ find $a_{12}$, which is the element on the first row and second column of $A^{-1}$.

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. $\frac{1}{4}$

D. $-\frac{1}{4}$

E. $\frac{1}{6}$

**8.** One solution of the differential equation

$$y^{(4)} + 2y^{(3)} + y'' + 2y' = 0$$

is $y_1(x) = e^{-2x}$. Then the general solution is

A. $c_1 \sin x + c_2 \cos x + c_3 e^{-2x} + c_4$

B. $c_1 \sin x + c_2 \cos x + c_3 e^{x} + c_4 e^{-x}$

C. $c_1 \sin x + c_2 \cos x + c_3 e^{2x} + c_4 e^{-2x}$

D. $c_1 \sin x + c_2 e^{x} + c_3 e^{-x} + c_4$

E. $c_1 \sin x + c_2 \cos x + c_3 e^{2x} + c_4 x$

**9.** Let $y$ be the solution of the initial value problem

$$x^2 y'' - 2xy' = 0, \quad y(1) = 0, \quad y'(1) = 1.$$

Find $y(2)$.

A. $y(2) = \frac{1}{4}$

B. $y(2) = 2e$

C. $y(2) = 3$

D. $y(2) = \frac{7}{3}$

E. $y(2) = \frac{5}{4}$

**10.** Evaluate the Wronskian determinant $W(x)$ of four functions

$$(\sin x, \cos x, \sin 2x, \cos 2x)$$

at the point $x = 0$.

A. $W(0) = 0$

B. $W(0) = 18$

C. $W(0) = 2$

D. $W(0) = 10$

E. $W(0) = -2$

**This test has one more page**

**11.** Find the general solution of the differential equation

$$y^{(3)} - 8y = 0.$$

A. $y(x) = c_1 e^{2x} + c_2 x e^{-2x} + c_3 x^2 e^{-2x}$

B. $y(x) = c_1 e^{2x} + c_2 \cos(2x) + c_3 \sin(2x)$

C. $y(x) = c_1 e^{2x} + c_2 e^{x} \cos(\sqrt{3}x) + c_3 e^{x} \sin(\sqrt{3}x)$

D. $y(x) = c_1 e^{2x} + c_2 e^{-x} \cos(2x) + c_3 e^{-x} \sin(2x)$

E. $y(x) = c_1 e^{2x} + c_2 e^{-x} \cos(\sqrt{3}x) + c_3 e^{-x} \sin(\sqrt{3}x)$