MA 262, Spring 2018, Midterm 2
Version 01 (Green)

INSTRUCTIONS

1. **Switch off your phone upon entering the exam room.**
2. **Do not open the exam booklet until you are instructed to do so.**
3. **Before you open the booklet, fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
4. **MARK YOUR TEST NUMBER ON THE SCANTRON**
5. Once you are allowed to open the exam, make sure you have a complete test. There are 6 different test pages with a total of 12 problems, plus this cover page.
6. Do any necessary work for each problem on the space provided, or on the back of the pages of this booklet. Circle your answers in the booklet.
7. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

RULES REGARDING ACADEMIC DISHONESTY

1. Do not leave the exam during the first 20 minutes of the exam.
2. No talking. Do not seek or obtain any kind of help from anyone to answer the problems on the exam. If you need assistance, consult an instructor.
3. Do not look at the exam of another student. You may not compare answers with other students until your exam is finished and turned in, and then only after you have left the room.
4. Your bags must be closed throughout the exam period.
5. Notes, books, calculators and phones must be in your bags and cannot be used.
6. Do not handle phones or cameras or any other electronic device until you have finished and turned in your exam, and then only if you have left the room.
7. When time is called, all students must put down their writing instruments immediately.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME ______________________________

STUDENT SIGNATURE ______________________________

STUDENT PUID ______________________________

SECTION NUMBER ______________________________

RECITATION INSTRUCTOR ______________________________

1. (8 points) For what value of k is the vector $(-1, 2, 1, 1)$ in the span of $(1, 2, 1, -1)$ and $(2, 2, 1, k)$?

   A. 2

   B. 1

   C. 0

   D. -1

   E. -2

2. (8 points) Which of the following sets S are subspaces of the corresponding vector spaces V?

   (i) $V = \mathbb{R}^3$ and S is the set of vectors (x, y, z) satisfying $x + 2y - z = 0$.

   (ii) $V = M_2(\mathbb{R})$ and S is the set of all 2×2 invertible matrices.

   (iii) $V = P_2$ (the set of polynomials of degree at most 2) and S is the set of polynomials of the form $bx + c$.

   (iv) V is the set of all twice differentiable functions and S is the set of the functions satisfying the differential equation $y'' - 3y' + 2y = 0$.

   A. (i) and (iv) only.

   B. (i) and (iii) only.

   C. (iii) and (iv) only.

   D. (i) (iii) and (iv).

   E. (i) (ii) (iii) and (iv).

3. (8 points) Determine all values of t such that the following three polynomials

$$x^2 + tx, \quad tx + 2, \quad tx^2 - 4$$

form a basis of P_2.

A. $t \neq 0, 2$

B. $t = 0, -2$

C. $t \neq 0, -1$

D. $t = 0, 1$

E. $t \neq 0, -3$

4. (8 points) Let A be a 3×6 matrix, and let B denote the reduced row echelon matrix of A. Which of the following statements is TRUE?

A. If the columns 1,3 of B have leading 1's then the columns 1,3 of A span $\mathbb{R}^3$.

B. If the number of non-zero rows of B equals 2, then one can pick 2 columns of A that form a basis of the column space of A.

C. If B has exactly 2 leading 1's, then the dimension of null space of A is 2.

D. If the number of non-zero rows of B equals 3, then the number of non-zero columns of B equals 3.

E. None of the above.

5. (8 points) T is a linear transformation from P_2 to P_2, where P_2 is the space of all polynomials of degree no more than 2, and

$$T(x^2 - 1) = 2x^2 + x - 3, \quad T(2x) = 4x, \quad T(3x + 2) = 2x + 6.$$

What is $T(x^2)$?

A. $2x^2 - x$

B. $x^2 - x$

C. $2x - 1$

D. $-2x + 3$

E. $2x$

6. (8 points) Let $T\colon \mathbb{R}^5 \to \mathbb{R}^4$, $T(\mathbf{x}) = A\mathbf{x}$ where

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & -3 \\ 1 & 1 & 1 & 2 & -5 \\ 2 & 3 & 1 & 4 & -5 \\ 3 & 4 & 2 & 5 & -8 \end{bmatrix}.$$

If $a = \dim(\mathrm{Rng}(T))$ and $b = \dim(\mathrm{Ker}(T))$, then $a + 2b$ equals

A. 5

B. 6

C. 7

D. 8

E. 9

7. (8 points) Which of the following statements is TRUE about matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}?$$

A. The sum of the eigenvalues of A is 1.

B. A has an eigenspace of dimension 2.

C. A is defective.

D. The vector $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is an eigenvector to the eigenvalue 2.

E. The vector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is an eigenvector to the eigenvalue 2.

8. (8 points) If y is the solution of the initial value problem

$$y'' - 5y' + 6y = 0, \qquad y(0) = 0, \quad y'(0) = 1$$

then $y(1) =$

A. $3e^2 - 3e^3$

B. $2e^3 - e^2$

C. $4e^3 - 2e^2$

D. $e^3 + e^2$

E. $e^3 - e^2$

9. (8 points) $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 2 & 4 & 0 & 7 \\ 1 & 2 & 1 & 0 & 5 & 0 \end{bmatrix}$, which of the following is a basis of the column space of A?

A. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \\ 1 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 2 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 4 \\ 0 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 4 \\ 0 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ 3 \\ 7 \\ 0 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 3 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 0 \\ 5 \end{bmatrix} \right\}$

10. (8 points) Find the general solution of

$$y^{(4)} - 5y'' + 4y = 0$$

A. $y(t) = c_1 e^t + c_2 e^{-t} + c_3 \cos 2t + c_4 \sin 2t$

B. $y(t) = c_1 e^t + c_2 e^{-t} + c_3 e^{2t} + c_4 e^{-2t}$

C. $y(t) = c_1 e^t + c_2 e^{-t} + c_3 \cos t + c_4 \sin t$

D. $y(t) = c_1 e^{2t} + c_2 e^{-2t} + c_3 t e^{2t} + c_4 t e^{-2t}$

E. $y(t) = c_1 e^t + c_2 e^{-t} + c_3 t e^t + c_4 t e^{-t}$

11. (10 points) Which of the following statements are always true?

(i) If A is an $n \times n$ matrix and A is nonsingular, then the columns of A must form a basis for $\mathbb{R}^n$.

(ii) Any set of linearly independent vectors of a vector space V must span V.

(iii) The annihilator of $F(x) = xe^{ax}$ is $A(D) = (D - a)^2$.

(iv) If a matrix A has a repeated eigenvalue, then A is defective.

A. (i) and (ii) only

B. (i) and (iii) only

C. (i), (iii), and (iv)

D. (i), (ii), and (iii)

E. (i), (ii), (iii), and (iv)

12. (10 points) Determine a suitable form of the particular solution $y_p(t)$ of the equation

$$y''' - y' = 2t + 4\sin t + e^{-t}.$$

A. $y_p(t) = (At + B)t^2 + C\sin t + D\cos t + Ete^{-t}$

B. $y_p(t) = (At + B)t + C\sin t + D\cos t + Ee^{-t}$

C. $y_p(t) = (At + B)t + C\sin t + D\cos t + Ete^{-t}$

D. $y_p(t) = At + B + C\sin t + D\cos t + Ete^{-t}$

E. $y_p(t) = At + B\sin t + C\cos t + De^{-t}$