Student: ____________________ **Date:** ____________________	**Instructor:** Antonio PurdueMath **Course:** MA262 Fall2020 Coordinator course	**Assignment:** Online-Midterm 2

1. Find a linear homogeneous constant-coefficient equation with the given general solution.

$$\left(A + Bx + Cx^2\right)e^{3x}$$

Choose the correct answer below.

- **A.** $y^{(4)} - 12y^{(3)} + 54y'' - 108y' + 81y = 0$
- **B.** $y' + 3y = 0$
- **C.** $\mathbf{y^{(3)} - 9y'' + 27y' - 27y = 0}$
- **D.** $y'' - 6y' + 9y = 0$

2. Three vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are given. If they are linearly independent, show this; otherwise, find a nontrivial linear combination of them that is equal to the zero vector.

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 3 \\ -2 \\ 11 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 5 \\ -7 \\ 17 \end{bmatrix}$$

Select the correct answer below, and fill in the answer box(es) to complete your choice.

- **A.**

  The vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are linearly independent. The augmented matrix $\begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{0} \end{bmatrix}$ has an echelon for

  (Type an integer or simplified fraction for each matrix element.)

- **B.** The vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are linearly dependent, because $4\mathbf{v}_1 + (\_\_\_\_\_)\mathbf{v}_2 + (\_\_\_\_\_)\mathbf{v}_3 = \mathbf{0}$.
  (Type integers or fractions.)

3. Find both a basis for the row space and a basis for the column space of the given matrix **A**.

$$\begin{bmatrix} 4 & 2 & -5 & 7 \\ 12 & 1 & 6 & 18 \\ 4 & -3 & 16 & 4 \\ 8 & -11 & 53 & 5 \end{bmatrix}$$

A basis for the row space is $\left\{ \begin{bmatrix} 4 & 2 & -5 & 7 \end{bmatrix}, \begin{bmatrix} 0 & -5 & 21 & -3 \end{bmatrix} \right\}$.

(Use a comma to separate matrices as needed.)

A basis for the column space is $\left\{ \begin{bmatrix} 4 \\ 12 \\ 4 \\ 8 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -3 \\ -11 \end{bmatrix} \right\}$.

(Use a comma to separate matrices as needed.)

4. Find a general solution to the differential equation given below. Primes denote derivatives with respect to t.

$$56y'' - 5y' - 6y = 0$$

A general solution is $y(t) = c_1 e^{\frac{3}{8}t} + c_2 e^{-\frac{2}{7}t}$.

5. Let a subset W be the set of all vectors in $\mathbb{R}^4$ such that $x_1 + 4x_2 + 5x_3 + 6x_4 = 0$. Apply the theorem for conditions for a subspace to determine whether or not W is a subspace of $\mathbb{R}^4$.

According to the theorem of conditions for a subspace, the nonempty subset W of the vector space V is a subspace of V if and only if it satisfies the following two conditions:

(i) If **u** and **v** are vectors in W, then **u** + **v** is also in W.
(ii) If **u** is in W and c is a scalar, then the vector c**u** is also in W.

Select the correct choice below.

- ○ **A.** W is not a subspace of $\mathbb{R}^4$ because condition (ii) fails while condition (i) is satisfied.
- ◉ **B. W is a subspace of $\mathbb{R}^4$ because it satisfies both of the conditions.**
- ○ **C.** W is not a subspace of $\mathbb{R}^4$ because both conditions (i) and (ii) fail.
- ○ **D.** W is not a subspace of $\mathbb{R}^4$ because condition (i) fails while condition (ii) is satisfied.

6. Determine whether the given vectors **u** and **v** are linearly dependent or linearly independent.

$$\mathbf{u} = \begin{bmatrix} -6 \\ 6 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 6 \\ 6 \end{bmatrix}$$

Select the correct choice below and, if necessary, fill in the answer box(es) to complete your choice.

- ○ **A.** The vectors are linearly dependent because **v** is a scalar multiple of **u**, specifically **v** = ______ **u**. (Type an integer or a fraction.)
- ○ **B.** The vectors are linearly independent because **v** is a scalar multiple of **u**, specifically **v** = ______ **u**. (Type an integer or a fraction.)
- ○ **C.** The vectors are linearly dependent because the only solution to the vector equation a**u** + b**v** = **0** is a = ______ (Type integers or fractions.)
- ◉ **D. The vectors are linearly independent because the only solution to the vector equation au + bv = 0 is a = ______ (Type integers or fractions.)**

7. If A is a 3x5 matrix. Which of the following statements are correct?

I) There is only one 5x1 vector X such that AX=0.

II) There are infinitely many 5x1 vectors X such that AX = 0 .

III) If b is a an arbitrary 3x1 vector, one can always find a 5x1 vector X which satisfies the system AX = b.

IV) If b is a 3x1 vector and X is a 5x1 vector such that AX = b, then there are infinitely many 5x1 vectors Y that satisfy AY = b.

- ○ **A.** Only I is true, II, III and IV are false
- ○ **B.** I and III are true, II and IV are false
- ◉ **C. II and IV are true, I and III are false**
- ○ **D.** I and IV are true, II and III are false
- ○ **E.** Only II is true, I, III and IV are false

8. If $A=\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & 1 & 4 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 0 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ and if $C = [c_{ij}]$ is the matrix such that C A=B, then which of the following statements is correct about the matrix C and its element c_{ij} which is on the the i-th row and j-th column?

- ○ **A.** C is a 2x3 matrix and $c_{11} = 2$
- ◉ **B. C is a 2x 3 matrix and $c_{22} = -19$**
- ○ **C.** C is a 3x2 matrix and $c_{11} = 0$
- ○ **D.** C is a 2x3 matrix and $c_{12} = -1$
- ○ **E.** C is a 3x2 matrix and $c_{13} = 6$

9. Which of the following are true statements about the solutions of the system

$x_1 - x_2 + 3x_3 = 1$

$x_1 + (b-1)x_2 + x_3 = 2$

$x_1 + x_2 + x_3 = b,$

where b is a constant?

I) The system has no solutions if b=2.

II) The system has only one solution if b is not equal to 2.

III) The system has infinitely many solutions if b=2.

IV) The system has no solutions if b=4.

- **A.** III and IV are true, I and II are false
- **B.** I and II are true, III and IV are false
- **C. II and III are true, I and IV are false**
- **D.** I and IV are true, II and III are false
- **E.** II is true, I, III and IV are false

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10. Find a basis for the indicated subspace of $\mathbf{R}^3$.

The line of intersection of the planes with equations $x - 5y + 3z = 0$ and $y = z$.

A basis for the indicated subspace of $\mathbf{R}^3$ is $\left\{ \begin{bmatrix} \mathbf{2} \\ \mathbf{1} \\ \mathbf{1} \end{bmatrix} \right\}$.

(Use a comma to separate vectors as needed.)

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11. If y(x) is the solution of the following initial value problem

$y''(x) - 5y'(x) + 6y(x) = 0$

$y(0) = 3$ and $y'(0) = 7$

then y(**ln** (2)) (that is, the value of y(x) when x = **ln** (2)) is equal to

- **A.** 12
- **B.** 10
- **C.** 8
- **D.** 18
- **E. 16**