MA 26200, Fall 2018, Exam 2
Version 01 (Green)

INSTRUCTIONS

(1) **Switch off your phone upon entering the exam room.**
(2) **Do not open the exam booklet until you are instructed to do so.**
(3) **Before you open the booklet, fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
(4) **MARK YOUR TEST NUMBER ON THE SCANTRON**
(5) Once you are allowed to open the exam, make sure you have a complete test. There are 6 different test pages, including this cover page.
(6) Do any necessary work for each problem on the space provided or on the back of the pages of this booklet. Circle your answers in the booklet.
(7) Use a # 2 pencil to transcribe your answers to the scantron.
(8) After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

RULES REGARDING ACADEMIC DISHONESTY

(1) Do not leave the exam during the first 20 minutes of the exam.
(2) Do not leave in the last 10 minutes of the exam.
(3) No talking. Do not seek or obtain any kind of help from anyone to answer the problems on the exam. If you need assistance, consult an instructor.
(4) Do not look at the exam of another student. You may not compare answers with other students until your exam is finished and turned in, and then only after you have left the room.
(5) Your bags must be closed throughout the exam period.
(6) Notes, books, calculators and phones must be in your bags and cannot be used.
(7) Do not handle phones or cameras or any other electronic device until you have finished and turned in your exam, and then only if you have left the room.
(8) When time is called, all students must put down their writing instruments immediately. You must remain in your seat while the TAs will collect the exam booklets and the scantrons.
(9) Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME ______________________________

STUDENT SIGNATURE ______________________________

STUDENT PUID ______________________________

SECTION NUMBER ______________________________

RECITATION INSTRUCTOR ______________________________

1. Let $A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 5 & 1 \\ 3 & 4 & -2 \end{bmatrix}$. Given that $det(A) = -10$, compute the (3,2) entry of A^{-1}.

A. $-\frac{1}{5}$
B. $\frac{1}{5}$
C. -5
D. $-\frac{1}{2}$
E. $\frac{1}{2}$

2. Which of the following are vector spaces?

I. $\{(x, y, z)$ in $\mathbb{R}^3$: $x + y - 2z = 4\}$.
II. The set of all polynomials in $\mathbb{P}_6$ satisfying $p(10) = 0$.
III. The set of all 2×2 matrices of the form $\begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$ where a and b are real numbers.

A. II and III only
B. I and II only
C. I and III only
D. All of these
E. II only

3. Given that the matrix $A = \begin{bmatrix} 1 & 2 & 2 & 3 & 8 \\ 0 & 0 & 1 & 1 & 3 \\ 1 & 2 & 2 & 3 & 12 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ is row-equivalent to the matrix $B = \begin{bmatrix} 1 & 2 & 2 & 3 & 8 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$, which of the following are true?

I. A basis for Nul A contains exactly one vector.

II. A basis for Col A is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ 3 \\ 1 \\ 0 \end{bmatrix} \right\}$.

III. A basis for Col A is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ 3 \\ 12 \\ 1 \end{bmatrix} \right\}$.

IV. The column space of A is $\mathbb{R}^4$.

A. II and III only
B. I and IV only
C. I only
D. II only
E. III only

4. What is the dimension of the subspace of $\mathbb{R}^4$ consisting of all vectors in $\mathbb{R}^4$ whose first and third entries are equal?

A. 1
B. 2
C. 3
D. 4
E. 0

5. Let $\mathbf{v_1} = \begin{bmatrix} 1 \\ -5 \\ 3 \\ 4 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 7 \\ 4 \\ -9 \\ -5 \end{bmatrix}$, and $\mathbf{v_3} = \begin{bmatrix} 4 \\ -7 \\ 2 \\ 5 \end{bmatrix}$. Which one of the following is a basis for Span $\{\mathbf{v_1},\ \mathbf{v_2},\ \mathbf{v_3}\}$?

A. $\left\{ \begin{bmatrix} 1 \\ -5 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 7 \\ 4 \\ -9 \\ -5 \end{bmatrix}, \begin{bmatrix} 4 \\ -7 \\ 2 \\ 5 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ -5 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 7 \\ 4 \\ -9 \\ -5 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 \\ -5 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 7 \\ 4 \\ -9 \\ -5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 3 \\ 0 \\ 0 \end{bmatrix} \right\}$

6. Which ones of the following statements are *always* true for an $m \times n$ matrix A?

I. The dimensions of the row space and the column space of A are always the same.

II. The sum of the dimensions of the rowspace of A and the null space of A equals the number of rows of A.

III. The dimension of the null space of A is the number of columns of A that are *not* pivot columns.

A. I only.
B. III only.
C. I and III.
D. I and II.
E. All of them.

7. Find the value of h in the matrix A below so that the dimension of the eigenspace for $\lambda = 5$ is 2:

$$A = \begin{bmatrix} 5 & -2 & 6 \\ 0 & 1 & h \\ 0 & 0 & 5 \end{bmatrix}$$

A. $h = 6$
B. $h = 12$
C. $h = 3$
D. No values of h
E. All values of h

8. If the determinant of $\begin{bmatrix} r & s & t \\ l & m & n \\ u & v & w \end{bmatrix}$ equals 2, what is the determinant of

$$\begin{bmatrix} r+2l & 3u & l \\ s+2m & 3v & m \\ t+2n & 3w & n \end{bmatrix}?$$

A. 54
B. 6
C. -54
D. -6
E. -108

9. Suppose a 2×2 real matrix A has a complex eigenvalue $\lambda_1 = 2 + i$ with a corresponding eigenvector $\mathbf{v}_1 = \begin{bmatrix} i \\ 1 \end{bmatrix}$, where $i = \sqrt{-1}$. Which one of the following statements is true?

A. $\lambda_2 = 2 - i$ is also an eigenvalue of A and $\mathbf{v}_2 = \begin{bmatrix} -i \\ 1 \end{bmatrix}$ is a corresponding eigenvector

B. $\lambda_2 = 2 - i$ is also an eigenvalue of A and $\mathbf{v}_2 = \begin{bmatrix} i \\ 1 \end{bmatrix}$ is a corresponding eigenvector

C. $A - \lambda_1 I$ is invertible, where I is the 2×2 identity matrix

D. The eigenspace associated with $\lambda_1 = 2 + i$ has a basis $\left\{ \begin{bmatrix} i \\ 1 \end{bmatrix}, \begin{bmatrix} -i \\ 1 \end{bmatrix} \right\}$

E. A is singular

10. Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 4 & 1 \end{bmatrix}$. The three eigenvalues of A are

A. 0, 1, 1 (two repeated eigenvalues)

B. 1, −1, 3

C. 1, 1 (two repeated eigenvalues), 3

D. 1, 1, 1 (three repeated eigenvalues)

E. 0, 1, −1

11. Find the solution to the initial value problem

$$y'' + 2y' + 2y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$

A. $y(t) = e^t + e^{-t}$
B. $y(t) = \cos t + \sin t$
C. $y(t) = e^t \cos t + e^t \sin t$
D. $y(t) = e^{-t} \cos t - e^{-t} \sin t$
E. $y(t) = e^{-t} \cos t + e^{-t} \sin t$

12. Consider the differential equation

$$y'' - 6y' + 9y = 0.$$

Which one of the following statements is true?

A. The equation has only one linearly independent solution e^{3t}
B. The equation has only one linearly independent solution te^{3t}
C. The equation has two linearly independent solutions e^{3t} and te^{3t}
D. The equation has two linearly independent solutions t and e^{3t}
E. The equation has two linearly independent solutions t^3 and e^{3t}