MA 262, Fall 2017, Midterm 2
Version 01 (Green)

INSTRUCTIONS

(1) **Switch off your phone upon entering the exam room.**
(2) **Do not open the exam booklet until you are instructed to do so.**
(3) **Before you open the booklet, fill in the information below and us a # 2 pencil to fill in the required information on the scantron.**
(4) **MARK YOUR TEST NUMBER ON THE SCANTRON**
(5) Once you are allowed to open the exam, make sure you have a complete test. There are 6 different test pages, plus this cover page. There are 12 questions total.
(6) Do any necessary work for each problem on the space provided or on the back of the pages of this booklet. Circle your answers in the booklet.
(7) Use a # 2 pencil to transcribe your answers to the scantron.
(8) After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

RULES REGARDING ACADEMIC DISHONESTY

(1) Do not leave the exam during the first 20 minutes of the exam.
(2) No talking. Do not seek or obtain any kind of help from anyone to answer the problems on the exam. If you need assistance, consult an instructor.
(3) Do not look at the exam of another student. You may not compare answers with other students until your exam is finished and turned in, and then only after you have left the room.
(4) Your bags must be closed throughout the exam period.
(5) Notes, books, calculators and phones must be in your bags and cannot be used.
(6) Do not handle phones or cameras or any other electric device until you have finished and turned in your exam, and then only if you have left the room.
(7) When time is called, all students must put down their writing instruments immediately. Come to the front of the room to turn in your test and scantron.
(8) Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME ______________________________

STUDENT SIGNATURE ______________________________

STUDENT PUID ______________________________

SECTION NUMBER ______________________________

RECITATION INSTRUCTOR ______________________________

(1) Of the following statements, determine which ones are correct.

(I) The set of solutions to the differential equation $y'' + 2y' + y + x = 0$ is a subspace of the vector space of all twice differentiable functions.

(II) The set of polynomials $p(x) = ax^2 + bx + c$ that satisfy $p(3) = 0$ is a subspace of the vector space P_2 of all polynomials of degree two or less.

(III) If V is a vector space and W is a subset of V that contains the zero vector then W must be a subspace of V.

(a) (I) and (II)
(b) (II) and (III)
(c) (I) and (III)
(d) (II) only
(e) (III) only

(2) The Wronskian $W(f_1, f_2, f_3)$ of the functions $f_1(x) = e^{2x}$, $f_2(x) = e^{-x}$, $f_3(x) = e^{4x}$ is

(a) $-7e^{5x}$
(b) $-8e^{5x}$
(c) $-30e^{5x}$
(d) $5e^{5x}$
(e) $2e^{5x}$

(3) The vector (a, b, c) is in the span of the vectors $(2, 3, 1)$ and $(1, -1, 1)$ if and only if

(a) $4a + b + 5c = 0$

(b) $a + 5b = 4c$

(c) $a + 5b + 4c = 0$

(d) $5a + 4b = c$

(e) $4a - b = 5c$

(4) Of the following statements,

(I) The functions $\sin(x)$ and $\cos(x)$ are a basis for the solution space to the differential equation $y''' + y' = 0$.

(II) Any three independent vectors in $\mathbb{R}^3$ form a basis for $\mathbb{R}^3$.

(III) A basis for a vector space V always contains the zero vector of V.

the following can be said:

(a) (I) and (III) are true

(b) (II) and (III) are true

(c) (I) is true but (III) is false.

(d) (II) is true but (III) is false.

(e) Each statement (I), (II), (III) is false.

(5) The values of k for which the polynomials $\{x^2-kx, x^2+x+1, x^2+x-1\}$ are linearly DEPENDENT are

(a) $k=-1$ only
(b) $k=1$ only
(c) $k=1,-1$
(d) All values of k
(e) No values of k

(6) Let $T\colon M_2(\mathbb{R}) \to M_2(\mathbb{R})$ be the linear transformation that sends the matrix A to $AB-BA$ where B is the fixed matrix $B=\begin{pmatrix} 2 & 3 \\ 4 & 2 \end{pmatrix}$.

The dimension of the kernel of T is

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

(7) Given is the matrix $A = \begin{pmatrix} 5 & -4 \\ 8 & -7 \end{pmatrix}$. Count the number of correct statements:

(I) $\lambda = 5$ is an eigenvalue of A.

(II) $v = (1, 2)$ is an eigenvector for A.

(III) The vector $(1, 1)$ is an eigenvector to the eigenvalue $\lambda = -7$.

(IV) The eigenvalues to A are the numbers 2 and $1 + i$.

(a) 3

(b) 0

(c) 1

(d) 4

(e) 2

(8) A is a 3×3 matrix with characteristic polynomial $p(\lambda) = (\lambda - 2)^2(\lambda + 2)$.

Select the correct statement.

(a) A must be defective.

(b) The eigenspace to the eigenvalue -2 can have dimension 2.

(c) If $\vec{v}$ is an eigenvector for A to the eigenvalue 2 and if $\vec{w}$ is a second eigenvector for A to the eigenvalue 2 then v and w must be linearly dependent.

(d) If $\vec{v}$ is an eigenvector for A to the eigenvalue 2 and if $\vec{w}$ is a second eigenvector for A to the eigenvalue 2 then v and w must be linearly independent.

(e) There exists a matrix A with characteristic poynomial as stated for which all four statements above are false.

(9) Two linearly independent solutions to the differential equation $y'' - 2y' - 3y = 0$ are given by

(a) $y_1(x) = 4e^{-x}$, $y_2(x) = 3e^{-2x}$
(b) $y_1(x) = 5e^{x}$, $y_2(x) = -e^{3x}$
(c) $y_1(x) = 8e^{-x}$, $y_2(x) = -3e^{-3x}$
(d) $y_1(x) = 2e^{3x}$, $y_2(x) = 3e^{-x}$
(e) $y_1(x) = 2e^{2x}$, $y_2(x) = 3e^{3x}$

(10) Of the statements:

(I) If D_1 and D_2 are linear differential operators then $D_1 D_2$ is also a linear differential operator.
(II) If D_1 and D_2 are linear differential operators then $D_1 D_2 = D_2 D_1$.
(III) If $y_p^{(1)}$ and $y_p^{(2)}$ are two particular solutions to the linear differential equation $Ly = F$ then $y_p^{(1)} - y_p^{(2)}$ is a solution to the associated homogeneous differential equation.

Which statement is true:

(a) (III) is correct but not (II).
(b) None of the three statements is correct.
(c) All of the statements are correct.
(d) (I) is correct but not (III).
(e) (I) and (II) are correct but not (III).

(11) The solution to the initial value problem

$$y'' + 4y' + 4y = 0, \qquad y(0) = 0, \qquad y'(0) = 4$$

satisfies

(a) $y(1) = 7 - e^2$
(b) $y(-1) = -5 - e^2$.
(c) $y(1) = 4/e^2$
(d) $y(1) = 2$
(e) $y(1) = 2 - 2e^2 + 1/e$

(12) Of the following sets:

(I) $\{(x, y) \in \mathbb{R}^2 |\ x \geq 0\}$

(II) $\left\{ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid \det A = 1 \right\}$

(III) $\{(x, y, z) \in \mathbb{R}^3 |\ 5x + 2y = z\}$

(IV) $\{f \in C^2(I) |\ f'' + f = 0\}$

select all those that are subspaces.

(a) (II), (III) and (IV)
(b) (I) and (II)
(c) (III) and (IV)
(d) (II) and (III)
(e) (II) and (IV)