February 21, 2023

MA262 - EXAM I

Green Exam: TEST NUMBER **01**

Instructions

(1) **DO NOT OPEN THIS EXAM BOOKLET UNTIL TOLD TO DO SO**.

(2) Before you open this exam booklet, fill in the information below and use a #2 pencil to fill in the required information on your scantron.

(3) On your scantron, write your 10-digit PUID (starting with "00" from left to right) and your 4-digit Recitation section number (starting with "0" from left to right).

(4) This Green exam is TEST NUMBER **01**.

(5) Once you are allowed to open this booklet, check to make sure you have a complete exam. There are **9** different exam pages including this cover page.

(6) Do any necessary work for each problem in the space provided or on the back of the pages of this exam booklet. No extra paper is allowed. **Circle** your answers in this exam booklet in case of a lost scantron.

(7) There are **11** problems, each worth 9 points and everyone gets 1 point. The maximum possible score is 100. No partial credit will be given.

(8) After you finish your exam, hand in both your scantron and your exam booklet to your instructor, your TA, or one of the proctors.

(9) You may not leave the exam room during the first 20 minutes of the exam. If you do not finish your exam within the first 50 minutes, you must wait until the end of the exam period to leave the room.

Academic Honesty

- Do not seek or obtain any assistance from anyone to answer questions on this exam.
- Do not talk during the exam. Any questions should be directed to your instructor or TA.
- Do not consult notes, books, or calculators during the exam. Do not handle phones, cameras, or any other electronic devices until after you have finished your exam, handed it in to your instructor, your TA or proctor, and left the room.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty may include an **F** in this course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the above statements regarding academic honesty :

Student Name _____ **PUID** _____

Student Signature _____

Recitation Section # _____ **TA Name** _____

1. Find the explicit solution to this initial value problem: $\dfrac{dy}{dx} + 4xy^3 = 0, \;\; y(1) = 1.$

A. $y = \dfrac{1}{\sqrt{2x^2 - 1}}$

B. $y = \dfrac{1}{(2x^2 - 1)^{\frac{1}{3}}}$

C. $y = \dfrac{1}{\sqrt{4x^2 - 3}}$

D. $y = \dfrac{x}{\sqrt{4x^2 - 3}}$

E. $y = \dfrac{1}{\sqrt{3x^2 - 2}}$

2. If $y(x)$ is the solution to the initial value problem $\begin{cases} xy' - 2y = x^3 e^x \\ y(1) = 0 \end{cases}$,

then $y(2) =$

A. $2e^2 - 2e$

B. $4e^2 - e$

C. $2e$

D. $4e^2 - 4e$

E. $\frac{1}{2}\left(e^2 - e\right)$

3. An object initially at 40 °F is placed in the air with the air temperature fixed. Suppose the temperature $T(t)$ of the object at time $t \geq 0$ satisfies the differential equation

$$\frac{dT}{dt} = 2(T - 32).$$

What is the temperature (in °F) of the object at time $t = 1$?

A. $32 + 6e$

B. $32 + 8e^2$

C. 32

D. $40 + e^2$

E. $32 + 8e^{-2}$

4. Find the general solution of the differential equation

$$\frac{dy}{dx} = 1 + \left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2, \quad x > 0.$$

A. $y = x \tan(\ln x + C)$

B. $y = x \tan(\ln x) + Cx$

C. $y = x \ln(\ln x + C)$

D. $y^2 = x^2 \tan(x + C)$

E. $y^2 = x^3 + Cx$

5. A large tank initially contains 40g of salt in 20L of water. A solution containing 3g/L of salt flows into the tank at a rate of 5L/min, and the well stirred mixture flows out at the rate of 2L/min. Which of the following describes the amount of salt $x(t)$ in the tank after t minutes? (g is grams; L is liters)

A. $\dfrac{dx}{dt} = 15 - \dfrac{1}{20}\,x, \quad x(0) = 40$

B. $\dfrac{dx}{dt} = 6 - \dfrac{2}{3t}\,x, \quad x(0) = 20$

C. $\dfrac{dx}{dt} = 6 - \dfrac{1}{10}\,x, \quad x(0) = 40$

D. $\dfrac{dx}{dt} = 15 - \dfrac{2}{20+3t}\,x, \quad x(0) = 40$

E. $\dfrac{dx}{dt} = 6 - \dfrac{2}{10+3t}\,x, \quad x(0) = 40$

6. Consider the following autonomous differential equation

$$\frac{dx}{dt} = x^2 - 5x + 6\,.$$

Which of one of the following statements is correct?

A. $x = 2$ is an **unstable** critical point and $x = 3$ is a **stable** critical point

B. $x = 2$ is a **stable** critical point and $x = 3$ is an **unstable** critical point

C. Both $x = 2$ and $x = 3$ are **stable** critical points

D. $x = 2$ is a **stable** critical point and $x = 3$ is a **semistable** critical point

E. $x = 2$ and $x = 3$ are **semistable**

7. Find an implicit solution of

$$\left(2xy + 2xy^2 + 1\right) + \left(x^2 + 2x^2y + 2y\right)\,\frac{dy}{dx} = 0\,.$$

A. $x^2y + 2x^2y^2 + y^2 = C$

B. $x^2y + x^2y^2 + x = C$

C. $x^2y + x^2y^2 + x + y^2 = C$

D. $2x^2y + x^2y^2 + 2x = C$

E. $x^2y + x^2y^2 + 2x = C$

8. Which of the following three statement(s) is/are TRUE?

(I) The function $y = x^2$ is a solution to $y' = 2\left(\frac{y}{x}\right)$.

(II) The differential equation $y^3y' + 2x^4y^4 = e^x$ is a Bernoulli differential equation.

(III) The substitution $v = x + y - 1$ will transform the equation $y' = (x + y - 1)^2$ into a Separable differential equation.

A. (I), (II), and (III) are TRUE

B. Only (II), and (III) are TRUE

C. Only (I) and (III) are TRUE

D. Only (I), (II) are TRUE

E. Only (I) is TRUE

9. Consider the linear system

$$\begin{aligned} x_1 + 2x_2 - x_3 &= a \\ 3x_1 - x_2 + 3x_3 &= 2 \\ 4x_1 + x_2 + 2x_3 &= b. \end{aligned}$$

Which of the following statement(s) is/are TRUE?

(I) The linear system is consistent for all values of a and b.

(II) The linear system has infinitely many solutions when $b = a + 2$.

(III) There is at least one choice of a and b for which the linear system has exactly one solution.

A. Only (I) is TRUE

B. Only (II) is TRUE

C. Only (III) is TRUE

D. Only (I) and (II) are TRUE

E. Only (II) and (III) are TRUE

10. Consider the linear system

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & k \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}.$$

Find k so that the system is inconsistent.

A. $k = 4$

B. $k = -4$

C. $k = 0$

D. $k = 2$

E. No value of k

11. Find the solutions $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ of this linear system:

$$\begin{aligned} x_1 - x_2 &= -2 \\ -x_1 - 5x_2 + 6x_3 &= 2 \\ -x_1 - 2x_2 + 3x_3 &= 2 \end{aligned}$$

A. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$

B. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} + s \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$ for all $s, t \in \mathbb{R}$

C. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + s \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$ for all $s, t \in \mathbb{R}$

D. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}$ for all $t \in \mathbb{R}$

E. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ for all $t \in \mathbb{R}$