MA 262, Spring 2019, Midterm 1
GREEN 01

INSTRUCTIONS

1. **Switch off your phone upon entering the exam room.**
2. **Do not open the exam booklet until you are instructed to do so.**
3. **Before you open the booklet, fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
4. **MARK YOUR TEST NUMBER ON THE SCANTRON**
5. Once you are allowed to open the exam, make sure you have a complete test. There are 6 different test pages with a total of 12 problems, plus this cover page.
6. Do any necessary work for each problem on the space provided, or on the back of the pages of this booklet. Circle your answers in the booklet.
7. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

RULES REGARDING ACADEMIC DISHONESTY

1. Do not leave the exam during the first 20 minutes of the exam.
2. No talking. Do not seek or obtain any kind of help from anyone to answer the problems on the exam. If you need assistance, consult an instructor.
3. Do not look at the exam of another student. You may not compare answers with other students until your exam is finished and turned in, and then only after you have left the room.
4. Your bags must be closed throughout the exam period.
5. Notes, books, calculators and phones must be in your bags and cannot be used.
6. Do not handle phones or cameras or any other electronic device until you have finished and turned in your exam, and then only if you have left the room.
7. When time is called, all students must put down their writing instruments immediately.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME ____________________

STUDENT SIGNATURE ____________________

STUDENT PUID ____________________

SECTION NUMBER ____________________

RECITATION INSTRUCTOR ____________________

1. (8 points) If $(x^2+3)\frac{dy}{dx} = xy$ and $y(1) = 8$, what is $y(3)$?

A. 12

B. 24

C. $4\sqrt{3}$

D. $8\sqrt{3}$

E. $6+2\sqrt{3}$

2. (8 points) For $x > 0$, the solution to the equation

$$\left(\frac{2y}{x} + 2x\right) + (2\ln(x) - 3)\, y' = 0$$

is given implicitly by

A. $y(2\ \ln(x) - 3) + x^2 = c$

B. $y(2\ \ln(x) + 3) - 3x^2 = c$

C. $\ln(x) - x^3 - 3y + c = 0$

D. $y(\frac{2}{x} + \frac{3}{2}) + x^2 + c = 0$

E. $\frac{1}{x} + x^2 - y^2 + c = 0$

3. (8 points) On the domain $(x > 0)$ consider the equation:

$$(x - 8y)\,dx + 4x\,dy = 0.$$

Its general solution is given by

A. $y = -x^4 \ln(x) + c\,x$

B. $y = \frac{x}{4} + c\,x^2$

C. $y = \frac{x}{2} \ln(x) + \frac{c}{x}$

D. $y = 4x^3 \ln(x) + c\,x$

E. $y = -x^3 + c\,x$

4. (8 points) Let y be the solution of the following equation:

$$t^2 y' + 4ty - y^3 = 0, \qquad y(1) = \frac{3}{\sqrt{2}}$$

Then $y(2) =$

A. $\sqrt{1/5}$

B. $\sqrt{10}$

C. $3\sqrt{6}$

D. 3

E. $3\sqrt{2}$

5. (8 points) Let

$$A = \begin{bmatrix} 1 & 4 & 7 & 10 \\ 2 & 5 & 8 & 11 \\ 3 & 6 & 9 & 12 \end{bmatrix}.$$

Which of the following collections of vectors is linearly independent?

A. The first three column vectors of the matrix A.

B. The last three column vectors of the matrix A.

C. The column vectors of the matrix A^T.

D. The column vectors of the matrix A.

E. The first two column vectors of the matrix A.

6. (8 points) A tank contains a mixture of 1 m^3 of water and 8 g of salt. Water containing a salt concentration of 4 g of salt per m^3 of water flows into the tank at a rate of $\frac{1}{4}$ m^3/min, and the mixture in the tank flows out at the same rate. We call $Q(t)$ the quantity of salt at time t in the tank. In order to have $Q(t) = 5$ g, we have to wait for

A. 3 min

B. $2\ln(7)$ min

C. $4\ln(4)$ min

D. $2\ln(4)$ min

E. $\frac{1}{2}\ln(2)$ min

7. (8 points) Given the vectors $\mathbf{v}_1 = \begin{bmatrix} -1 \\ 4 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 1 \\ h \\ 1 \end{bmatrix}$, for what value of h is $\mathbf{b}$ in the span of $\mathbf{v}_1$ and $\mathbf{v}_2$?

A. $h = 0$

B. $h = 1$

C. $h = 2$

D. $h = 3$

E. $h = 4$

8. (8 points) Consider the linear system

$$\begin{aligned} x_1 + x_3 &= 2 \\ x_1 + 2x_2 - x_3 &= 4 \\ 3x_1 + x_2 + 2x_3 &= 7 \end{aligned}$$

Which of the following gives the solutions of the system in parametric form?

A. $\mathbf{x} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + \mathrm{t} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ with any $t \in \mathbb{R}$

B. $\mathbf{x} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + \mathrm{t} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ with any $t \in \mathbb{R}$

C. $\mathbf{x} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + \mathrm{t} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ with any $t \in \mathbb{R}$

D. $\mathbf{x} = \mathrm{s} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + \mathrm{t} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ with any $s, t \in \mathbb{R}$

E. $\mathbf{x} = \mathrm{s} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + \mathrm{t} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ with any $s, t \in \mathbb{R}$

9. (8 points) Find all values of h for which the linear system has no solutions.

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 4 \\ x_1 - x_2 + x_3 &= 5 \\ 3x_1 + x_2 + hx_3 &= -1 \end{aligned}$$

A. $h = 0$

B. $h = 1$

C. $h = 2$

D. $h = 3$

E. $h = 4$

10. (8 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation for which

$$T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\2\\3\end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}4\\5\\6\end{bmatrix}, \text{then} \quad T\left(\begin{bmatrix}3\\4\end{bmatrix}\right) =$$

A. $\begin{bmatrix}0\\3\\2\end{bmatrix}$

B. $\begin{bmatrix}0\\2\\4\end{bmatrix}$

C. $\begin{bmatrix}1\\3\\5\end{bmatrix}$

D. $\begin{bmatrix}1\\-1\\0\end{bmatrix}$

E. $\begin{bmatrix}0\\3\\6\end{bmatrix}$

11. (10 points) Let A be an 6×5 matrix, and let B denote the reduced row echelon matrix of A. Which of the following statements are true?

(i) If B has 5 pivot columns, then the column vectors of A are always linearly independent.

(ii) If B has only 4 pivot columns, then the column vectors of A are always linearly dependent.

(iii) If B has only 2 leading 1's, then $A\mathbf{x} = \mathbf{b}$ always has infinitely many solutions, where $\mathbf{b}$ is any vector in $\mathbb{R}^6$.

(iv) If the entries of the last row of B are all zero, then $A\mathbf{x} = \mathbf{0}$ always has some nontrivial solution.

A. (ii) and (iv) only

B. (i) and (ii) only

C. (i) and (iv) only

D. (i), (ii) and (iii)

E. (i), (ii) and (iv)

12. (10 points) Let

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{bmatrix}.$$

Find the sum of the elements of the third column in $A^{-1}B$.

A. 0

B. 1

C. 2

D. 3

E. 4