MA262 — EXAM I — FALL 2022 — OCTOBER 6, 2022
TEST NUMBER 01

INSTRUCTIONS:

1. **DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED TO DO SO**
2. Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.
3. **MAKE SURE YOU WRITE YOUR 10 DIGIT ID # AND YOUR TEST NUMBER ON YOUR SCANTRON. THIS IS TEST NUMBER 01.**
4. Once you are allowed to open the exam, make sure you have a complete test. **There are 8 different test pages including this cover page.**
5. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. No extra paper is allowed. Circle your answers on this test booklet.
6. There are eleven problems, each problem is worth 9 points and everyone gets one point. The maximum possible score is 100 points. No partial credit.
7. After you finish the exam, hand in your scantron and your test booklet to your professor, your TA or one of the proctors.

RULES REGARDING ACADEMIC DISHONESTY:

1. Do not leave the exam room during the first 20 minutes of the exam.
2. If you do not finish your exam in the first 50 minutes, you must wait until the end of the exam period to leave the room.
3. Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only your instructor.
4. Do not look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
5. Do not consult notes, books, calculators.
6. Do not handle phones or cameras, or any electronic device until after you have finished your exam, handed it in to your instructor and left the room.
7. After time is called, the students have to put down all writing instruments and remain in their seats, while the proctors will collect the scantrons and the exams.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: _____ ID NUMBER: _____

SIGNATURE: _____

RECITATION SEC. NUMBER _____ TA's NAME: _____

1. If $y\,\dfrac{dy}{dx} = \dfrac{y^2-1}{2x}$ and $y(1) = 2$, then $y(4) = ?$

A. $\sqrt{13}$

B. $\sqrt{7}$

C. $\sqrt{5}$

D. 7

E. $\sqrt{10}$

2. If $y(x)$ is the solution to the initial value problem $\begin{cases} (x+1)\,y' + 2y = \dfrac{2}{(x+1)} \\ y(0) = 1 \end{cases}$,

find $y(1)$.

A. $\dfrac{5}{9}$

B. $\dfrac{3}{4}$

C. $\dfrac{7}{9}$

D. $\dfrac{4}{3}$

E. 2

3. A tank initially contains 80 L (liters) of a solution with 15 gms of chlorine in it. To dilute the tank, pure water is pumped in at a rate of 8 L/min and the well-stirred mixture is pumped out at the same rate. If $x(t)$ is the number of grams of chlorine in the tank at time t, find the number of grams of chlorine in the tank after 10 minutes.

A. $\dfrac{15}{e}$ gms

B. $15\left(1-e^{-1}\right)$ gms

C. $15e^{-\frac{1}{8}}$ gms

D. 5 gms

E. $\dfrac{3}{2}$ gms

4. The general solution of

$$\frac{dy}{dx}=\frac{3x-y}{x+y}$$ is

A. $y^2+8xy-4x^2=C$

B. $y^2-3xy+3x^2=C$

C. $y^2+4xy+3x^2=C$

D. $y^2+2xy+3x^2=C$

E. $y^2+2xy-3x^2=C$

5. Which of the following statement(s) is/are **TRUE** ?

(I) The Existence and Uniqueness Theorem for first order linear differential equations guarantees that the solution to the initial value problem $\begin{cases} (x+1)y' = \left(\dfrac{\sin x}{x-2}\right) y + \cos x \\ y(1) = 3 \end{cases}$ will exist in the interval $-2 < x < 2$.

(II) $y = \sqrt{x}$ is a solution to $y\,\dfrac{dy}{dx} = \dfrac{1}{2}$.

(III) $\dfrac{dy}{dt} + 3y = t^2 y^2$ is a Bernoulli differential equation.

A. Only **(I)** and **(II)** are true

B. Only **(II)** and **(III)** are true

C. Only **(II)**

D. Only **(I)** and **(III)** are true

E. **(I)**, **(II)** and **(III)** are true

6. Which of the following is the implicit solution to the initial value problem

$$(e^x \sin y - 2y \sin x - 1)dx + (e^x \cos y + 2\cos x + 1)dy = 0,$$
$$y(0) = \pi?$$

A. $e^x \sin y + 2y \cos x + x - y = \pi$

B. $e^x \sin y + 2y \cos x + x - y = 0$

C. $e^x \sin y + 2y \cos x + y - x = \pi$

D. $e^x \sin y + 2y \cos x + y - x = 3\pi$

E. $e^x \sin y - 2y \cos x + y - x = 3\pi$

7. Consider the equation

$$\frac{dy}{dx} + \frac{1}{x}y = 3xy^2.$$

If $v = y^{-1}$, then v satisfies

A. $\dfrac{dv}{dx} + \dfrac{1}{x}v = -3x$

B. $\dfrac{dv}{dx} - \dfrac{1}{x}v = -3x$

C. $\dfrac{dv}{dx} + \dfrac{1}{x}v = 3x$

D. $\dfrac{dv}{dx} - \dfrac{1}{x}v = 3x$

E. $\dfrac{dv}{dx} + \dfrac{1}{2x}v = -3x$

8. Which of the following alternatives lists **all** stable equilibrium solutions to the differential equation

$$\frac{dy}{dx} = y(y-1)(y-2)?$$

A. $y(x) = 0$

B. $y(x) = 1$

C. $y(x) = 0$ and $y(x) = 2$

D. $y(x) = 0$ and $y(x) = 1$

E. $y(x) = 1$ and $y(x) = 2$

9. Which of the following are true statements about the matrix $A = \begin{bmatrix} 3 & 9 & 1 \\ 2 & 6 & 7 \\ 1 & 3 & -6 \end{bmatrix}$?

I. $E_0 = \begin{bmatrix} 1 & 3 & -6 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is a row echelon form of A.

II. $E_0 = \begin{bmatrix} 1 & 3 & -6 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ and $E_1 = \begin{bmatrix} 1 & 3 & -6 \\ 0 & 0 & 19 \\ 0 & 0 & 0 \end{bmatrix}$ are row echelon forms of A.

III. $E_0 = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is the reduced row echelon form of A.

IV. $E_0 = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ and $E_1 = \begin{bmatrix} 1 & 3 & -6 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ are both reduced row echelon forms of A.

A. Only I is true

B. Only II is true

C. Only II and III are true

D. Only IV is true

E. Only I, II and III are true.

10. Which of the following four statements about the system

$$\begin{aligned} x + 3y + z &= 1 \\ 3x + ky + z &= 3 \\ 2x + 6y + 7z &= b \end{aligned}$$

are true?

I. The system will have only one solution for any values of k and b.

II. If $k = 9$ and $b = 3$ this system will have no solutions

III. If $k = 9$ and $b = 2$ this system will have infinitely many solutions

IV. If $k = 7$ this system will have only one solution for any value of b

A. Only I is true

B. Only II and III are true

C. Only I and III are true

D. Only III and IV are true

E. Only II, III and IV are true

11. Let $A = \begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ and $C = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$ be 2×2 matrices such that

$$AB = \begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix} \text{ and } CA = \begin{bmatrix} 1 & 3 \\ 5 & 2 \end{bmatrix}.$$

Given that the inverse of the matrix A is equal to $A^{-1} = \begin{bmatrix} 1 & -3 \\ -1 & 4 \end{bmatrix}$, we can say that the elements b_{12} of the matrix B and c_{12} of the matrix C are

A. $b_{12} = c_{12} = -3$

B. $b_{12} = -2$ and $c_{12} = 12$

C. $b_{12} = -3$ and $c_{12} = -4$

D. $b_{12} = -3$ and $c_{12} = 9$

E. $b_{12} = 1$ and $c_{12} = 9$