MA 26100
FINAL EXAM Form 01
May 3, 2023

NAME ____________________ YOUR TA'S NAME ____________________

STUDENT ID # __________________ RECITATION TIME __________________

Be sure the paper you are looking at right now is GREEN! Write the following in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below the boxes on the scantron: **01**

You must use a #2 pencil on the scantron answer sheet. Fill in the following on your scantron and blacken the bubbles

1. Your name. If there aren't enough space for your name, fill in as much as you can.
2. Section number. If you don't know your section number, ask your TA.
3. Test/Quiz number: **01**
4. Student Identification Number: **This is your Purdue ID number with two leading zeros.**

There are **20** questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–20. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the scantron and the exam booklet when you are finished.

If you finish the exam before 2:50pm, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 1:20pm. If you don't finish before 2:50pm, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT SIGNATURE: __________________________________

1. Suppose $\langle a, b, c\rangle$ is a unit vector. Find an equation of the plane with normal vector $\langle a, b, c\rangle$ that contains (a, b, c).

A. $x = at$, $y = bt$, $z = ct$, where t is a real number.

B. $x = a$

C. $ax + by + cz = 1$

D. $ax + by + cz = 0$

E. $a^2x + b^2y + c^2z = 1$

2. Identify the quadric surface $16x^2 + 4y^2 - z^2 - 64x + 80 = 0$.

A. Hyperboloid of two sheets

B. Ellipsoid

C. Elliptic cone

D. Elliptic paraboloid

E. Hyperboloid of one sheet

3. A particle with initial velocity $\vec{v}(0) = \langle 1, -1, 1\rangle$ has acceleration $\vec{a}(t) = \langle 4t, 6t, 1\rangle$ Find the speed of the particle after time $t = 1$.

A. $\sqrt{13}$

B. $\sqrt{14}$

C. $\sqrt{15}$

D. 4

E. $\sqrt{17}$

4. A blue plane $B : x + 3y - 2z = 6$ and a yellow plane $Y : 2x + y + z = 3$ intersect in a green line G. A projectile at $P(1, -2, -1)$ starts moving in the direction $\vec{v} = \langle 1, 2, 1\rangle$. Which of the following statements is true?

A. The projectile intersects the blue plane first and then intersects the yellow plane

B. The projectile intersects the yellow plane first and then intersects the blue plane

C. The projectile intersects the green line first

D. The projectile intersects the blue plane and does not intersect the yellow plane

E. The projectile intersects the yellow plane and does not intersect the blue plane

5. Find an equation of the tangent plane to the paraboloid

$$z = 1 - \frac{1}{10}\left(x^2 + 4y^2\right) ,$$

at the point $(1, 1, 1/2)$.

A. $-\frac{1}{30}x + \frac{1}{5}y + z = \frac{2}{3}$
B. $\frac{4}{5}x + \frac{1}{5}y + z = \frac{3}{2}$
C. $\frac{4}{5}x + \frac{4}{5}y + z = \frac{21}{10}$
D. $\frac{1}{5}x + \frac{4}{5}y + z = \frac{3}{2}$
E. $\frac{1}{5}x + \frac{1}{5}y + z = \frac{9}{10}$

6. What value of c makes the function

$$f(x,y) = \begin{cases} \frac{x^2-y^2}{x-y} & \text{if} \quad x \neq y \\ cx & \text{if} \quad x = y \end{cases}$$

continuous?

A. 2
B. 0
C. 16
D. 1
E. Such a c does not exist

7. Let $f(x, y) = x^2 e^{x+y}$. Find a unit vector in the direction of most rapid decrease for f when $(x, y) = (1, 1)$.

A. $\langle 3e^2, e^2 \rangle$

B. $\frac{\langle 3,1 \rangle}{\sqrt{10}}$

C. $\frac{\langle -3e^2, -e^2 \rangle}{\sqrt{10}}$

D. $\frac{\langle -3,-1 \rangle}{\sqrt{10}}$

E. $\langle -3e^2, -e^2 \rangle$

8. Find the minimum value of $f(x, y) = x + y$ subject to the constraint $x^2 + 2y^2 = 6$

A. -2

B. -1

C. 3

D. 2

E. -3

9. The integral in spherical coordinates

$$\int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^{\csc\phi} \rho^2 \sin\phi \; d\rho d\phi d\theta$$

when converted into Cartesian coordinates becomes

A. $\int_0^1 \int_0^{1-x^2} \int_0^{\sqrt{x^2+y^2}} dz\; dy\; dx$

B. $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{x^2+y^2}} dz\; dy\; dx$

C. $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^1 dz\; dy\; dx$

D. $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \int_1^{\sqrt{x^2+y^2}} dz\; dy\; dx$

E. $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{x^2+y^2} dz\; dy\; dx$

10. The double integral $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} y^2(x^2+y^2)^3 \; dydx$ when converted to polar coordinates becomes

A. $\int_0^{\pi} \int_0^1 r^9 \sin^2\theta \; drd\theta$

B. $\int_0^{\frac{\pi}{2}} \int_0^1 r^9 \sin^2\theta \; drd\theta$

C. $\int_0^{\pi} \int_0^1 r^8 \sin\theta \; drd\theta$

D. $\int_0^{\frac{\pi}{2}} \int_0^1 r^8 \sin\theta \; drd\theta$

E. $\int_0^{\frac{\pi}{2}} \int_0^1 r^8 \sin^2\theta \; drd\theta$

11. Let $\vec{F}(x,y,z) = \langle 3x^2yz - 3y, x^3z - 3x, x^3y + 2z\rangle$. You may assume that $\vec{F}$ is conservative. Calculuate the work done by $\vec{F}$ moving an object along C where C is the line segment from (0,0,2) to (0,3,0).

A. 0

B. -9

C. 5

D. 12

E. -4

12. Evaluate

$$\oint_C x^2y\,dx + (y + y^2)\,dy,$$

where C is the boundary of the region enclosed by $y = x^2$ and $x = y^2$ oriented counter-clockwise.

A. $-\frac{2}{35}$

B. $-\frac{3}{35}$

C. $-\frac{4}{35}$

D. $-\frac{6}{35}$

E. $-\frac{8}{35}$

13. Compute the line integral

$$\int_C (2x + y)\, ds$$

where C is the line segment from (0,0) to (6,8).

A. 80

B. 120

C. 140

D. 100

E. 160

14. Suppose $\phi(x, y) = x^2y - y^2x$ is a scalar function. Let $\vec{F}(x, y)$ be the gradient field of $\phi(x, y)$. Compute $|\vec{F}(1, 2)|$

A. 5

B. 2

C. 3

D. 6

E. 8

15. The surface parametrized by $\vec{r}(u, v) = \langle 5\cos u \sin v, 5\sin u \sin v, 5\cos v\rangle$ is a

A. Plane

B. Ellipse

C. Sphere

D. Cylinder

E. Paraboloid

16. A normal vector to the surface $\mathbf{r}(u, v) = \langle \sin u, \cos u \sin v, \sin v\rangle$ at $(u, v) = (\pi/3, \pi/3)$ is

A. $\langle -3, -2, 1\rangle$

B. $\langle -1/8, 1/4, 1\rangle$

C. $\langle -3, 2, 1\rangle$

D. $\langle -3/8, 1/4, 2\rangle$

E. $\langle 3/8, 1/4, 2\rangle$

17. Let $\mathbf{F}(x, y, z) = \langle x^2y, y^2z, xy^2\rangle$, then curl $\mathbf{F}(3, 2, 1)$ is

A. $\langle 12, -4, -5\rangle$

B. $\langle 8, -4, -5\rangle$

C. $\langle 8, -12, 4\rangle$

D. $\langle 8, -4, -9\rangle$

E. $\langle 12, -12, 4\rangle$

18. Let S be an open surface in $\mathbb{R}^3$ with boundary curve C where C is a circle in the $xy-$plane with radius 1 and center $(0, 0, 0)$ and is oriented counterclockwise when viewed from above. Evaluate $\displaystyle\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y, z)$ is such that $\text{curl}(\vec{F}) = \langle \frac{1}{2}, -\frac{3}{2}, \frac{1}{4}\rangle$.

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. $-\frac{3\pi}{2}$

E. 2π

19. For which of the following vector fields is $\iint_S \vec{F} \cdot \vec{n}\, dS$ NOT equal to $\iiint_E dV$ if S is the closed boundary surface of the unit cube $E = \{(x, y, z) \mid 0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1\}$?

A. $\vec{F} = \langle x, x\cos z, e^{y^2 - x} \rangle$

B. $\vec{F} = \langle 3x, -y, -z \rangle$

C. $\vec{F} = \langle x, z\tan x, x\ln y \rangle$

D. $\vec{F} = \langle \frac{1}{3}x, \frac{1}{3}y, \frac{1}{3}z \rangle$

E. $\vec{F} = \langle xy^2, yz^2, zx^2 \rangle$

20. Find the outward flux of the vector field $\vec{F} = \langle \sin y, xz, 3z \rangle$ across the boundary of the space between two spheres of radii 1 and 2, both centered at the origin.

A. 84π

B. 24π

C. 28π

D. $\frac{28}{3}\pi$

E. 12π