MA 26100 Final Exam, Spring 2014

Name ______________________________

10 digit PUID number ______________________________

Recitation Instructor ______________________________

Recitation Section Number and Time ______________________________

Instructions: MARK TEST NUMBER 01 ON YOUR SCANTRON

1. Do not open this booklet until you are instructed to.
2. Fill in all the information requested above and on the scantron sheet. On the scantron sheet fill in the little circles for your name, section number and PUID.
3. This booklet contains 22 problems, each worth 9 points. There are 2 free points.
4. For each problem mark your answer on the scantron sheet and also **circle it in this booklet**.
5. Work only on the pages of this booklet.
6. Books, notes, calculators or any electronic device are not allowed during this test and they should not even be in sight in the exam room. You may not look at anybody else's test, and you may not communicate with anybody else, except, if you have a question, with your instructor.
7. You are not allowed to leave during the first 20 and the last 10 minutes of the exam.
8. When time is called at the end of the exam, put down your writing instruments and remain seated. The TAs will collect the scantrons and the booklets.

Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ________________________________

STUDENT SIGNATURE: ________________________________

1. The line through $(4, 0, -12)$ and parallel to the line $x = 2-3t$, $y = -1+4t$, $z = 3 + 6t$, intersects the xy plane at the point

A. $(-2, 8)$

B. $(\frac{3}{2}, -3)$

C. $(2, -8)$

D. $(-\frac{3}{2}, 3)$

E. $(12, -4)$

2. The plane containing the points $(1, 0, 2)$, $(2, -2, 3)$, and $(4, 5, 1)$ has a normal vector equal to:

A. $4\vec{i} + \vec{j} - 2\vec{k}$

B. $-17\vec{i} + 10\vec{j} + 18\vec{k}$

C. $-17\vec{i} - 10\vec{j} + 18\vec{k}$

D. $-3\vec{i} + 4\vec{j} + 11\vec{k}$

E. $-3\vec{i} - 4\vec{j} + 11\vec{k}$

3. The tangent line to the curve $\vec{r}(t) = \langle t^3, 1-t^2, 2t+1\rangle$ at the point $(8, -3, 5)$ is given by:

A. $\dfrac{x+8}{12} = \dfrac{y-3}{-4} = \dfrac{-z+5}{2}$

B. $\dfrac{x}{8} = \dfrac{y-1}{-3} = \dfrac{z-1}{5}$

C. $\dfrac{x-8}{6} = \dfrac{y+3}{-2} = z-5$

D. $\dfrac{x}{8} = \dfrac{y+1}{-3} = \dfrac{z+1}{5}$

E. $\dfrac{y+3}{-1} = \dfrac{z-5}{2}$

4. Let $\mathbf{F} = \langle ye^x, x^2, \cos(z)\rangle$. Then curl($\mathbf{F}$) is equal to:

A. $\langle ye^x, 0, -\sin(z)\rangle$
B. $ye^x - \sin(z)$
C. $\langle 0, 0, 0\rangle$
D. $\langle 0, 2x, 2x - e^x\rangle$
E. $\langle 0, 0, 2x - e^x\rangle$

5. The arclength of the curve $\vec{r}(t) = 9t\vec{i} + 4t^{3/2}\vec{j} + t^2\vec{k}$ for $0 \le t \le 1$ is:

A. 11/2

B. 10

C. 85/3

D. 11

E. 12

6. The acceleration of a particle at time t is $\vec{a}(t) = 6t\vec{i} - (\sin t)\vec{j} + e^t\vec{k}$. Its velocity at time $t = \pi$ is $\vec{v}(\pi) = 3\pi^2\vec{i} + \vec{j} + e^\pi\vec{k}$. What is its velocity at time $t = 0$?

A. $-\vec{j} + \vec{k}$

B. $2\vec{j} + \vec{k}$

C. $3\vec{j} + \vec{k}$

D. $3\pi^2\vec{i} - \vec{j}$

E. $3\pi^2\vec{i} + 2\vec{j}$

7. If z is given implicitly as a function of x and y by the equation $xyz + z^2 = 15$, find $\frac{\partial z}{\partial x}$ at the point $(2, 1, 3)$.

A. 0

B. -1

C. $-\frac{1}{4}$

D. $-\frac{3}{8}$

E. $-\frac{3}{4}$

8. Let $f(x, y) = \ln(2x - y)$. Using a linear approximation, the approximate value of $f(1.1, 0.9)$ is:

A. 0.3

B. -0.3

C. 0.1

D. -0.2

E. 0.2

9. Use the chain rule to find $\frac{dz}{dt}$ when $t = \frac{\pi}{4}$ if $z = (x^2 + 2y^2)^{3/2}$, $x = \sin t$, and $y = \cos t$:

A. $\frac{9}{2}\sqrt{\frac{3}{2}}$

B. $-\frac{3}{2}\sqrt{\frac{3}{2}}$

C. $\frac{9}{2}\sqrt{3}$

D. $-\frac{9}{2}\sqrt{3}$

E. $\frac{3}{2}\sqrt{3}$

10. The directional derivative of $f(x,y) = x^3e^{-2y}$ in the direction of greatest increase of f at the point $x = 1,\ y = 0$ is:

A. $3\vec{i}$

B. $3\vec{i} - 2\vec{j}$

C. 3

D. $\sqrt{5}$

E. $\sqrt{13}$

11. The function $f(x, y) = 2x^3 + 6xy + 3y^2$ has:

A. one local minimum and one local maximum

B. two saddle points

C. one local minimum and one saddle point

D. one local maximum and one saddle point

E. two local minima

12. Let D be the region in the x-y plane which lies inside the circle $x^2+y^2 = 4$. Then $\iint\limits_D y^2 \, dx \, dy$ is equal to:

A. 4
B. 8
C. $8\pi/3$
D. 4π
E. 8π

13. Let E be the solid region in the first octant bounded above by $f(x, y) = x^2+y$ and lying above the triangle with vertices $(0, 0, 0)$, $(1, 1, 0)$, and $(0, 1, 0)$. Find the volume of E.

A. 1/4

B. 5/12

C. 7/12

D. 1/2

E. 3/4

14. let S be the portion of the plane $x + \sqrt{2}y + z = 3$ inside the cylinder $x^2 + y^2 = 4$. Then $\iint\limits_S (x^2 + y^2)\, dS$ equals

A. $\int_0^{2\pi} \int_0^2 2r^3 dr d\theta$

B. $\int_0^{2\pi} \int_0^2 r^3 dr d\theta$

C. $\int_0^{2\pi} \int_0^2 r^2 dr d\theta$

D. $\int_0^{2\pi} \int_0^2 r^2 \sqrt{1 + 4r^2} dr d\theta$

E. $\int_0^{2\pi} \int_0^2 r^3 \sqrt{1 + 4r^2} dr d\theta$

15. Find the volume of the solid enclosed by $y = x^2 + z^2$ and $y = 8 - x^2 - z^2$.

A. 8π
B. 16π
C. 4π
D. 2π
E. None of the above

16. Let E be the solid in the first octant inside the sphere $x^2 + y^2 + z^2 = 4$ and between the planes $y = 0$ and $y = \sqrt{3}x$. Then

$$\iiint_E (x^2 + y^2 + z^2)\, dV$$

is equal to:

A. $(32\pi)/15$
B. $(8\pi)/15$
C. 16π
D. $(16\pi)/15$
E. $(4\pi)/3$

17. Let $\mathbf{F}(x, y, z) = \langle yz, xz, xy \rangle$, and let $\mathbf{r}(t)$ be a parameterization of any curve C going from $(1, 1, 0)$ to $(1, 1, 1)$. Then

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

is equal to:

A. -2
B. -1
C. 0
D. 1
E. 2

18. Let C be the closed curve given by the boundary of the rectangle with vertices $(-2, 0), (2, 0), (2, 1)$, and $(-2, 1)$ with positive orientation. Then

$$\int_C (y + \sinh(x^2 + 1))\ dx + (y \sin^2(y) + x^2)\ dy$$

is equal to (hint: Green's Theorem):

A. 0
B. 2
C. 4π
D. -2
E. -4

19. Let S be the part of the surface $z = x^2 + y^2$ below the plane $z = 1$. Let $\mathbf{n}$ be the upward unit normal on S and $\mathbf{F}(x, y, z) = \langle 2x, 2y, z\rangle$. Evaluate $\iint\limits_S \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S}$.

A. 2π
B. -2π
C. $3\pi/2$
D. $-3\pi/2$
E. 0

20. Let S be a surface given by $\mathbf{r}(u, v) = \langle uv, v, u\rangle$, where $u^2 + v^2 \leq 1$. Then the surface area of S is equal to:

A. $2\pi(4 - \sqrt{2})/6$
B. $2\pi(2\sqrt{2} - 1)/3$
C. $\pi(\sqrt{2})$
D. $4/3$
E. $(4\pi)/3$

21. Use Stokes' Theorem to evaluate

$$\iint_S \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S}$$

where $\mathbf{F}(x, y, z) = \langle e^{xy}, e^{xz}, x^2 z \rangle$, and S is the right half of the ellipsoid $4x^2 + y^2 + 4z^2 = 4$, with unit normal vector field oriented in the direction of the positive y-axis.

A. 0
B. -1
C. -2
D. 1
E. 2

22. Let $\mathbf{F}(x, y, z) = \langle xy, x, z \rangle$. Let E be the bounded solid region in the first octant bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 2$, $z = 0$, and $z = 3$, and let S be the boundary of E, with outward unit normal vector field. Use the Divergence Theorem to calculate

$$\iint_S \mathbf{F} \cdot d\mathbf{S}$$

A. 6
B. 8
C. 9
D. 12
E. 18