1. Find an equation of the plane that contains the point $(2, 1, 1)$ and the line

$$x = 1 + 3t, \quad y = 2 + t, \quad z = 4 + t.$$

A. $3x + y + z = 8$

B. $2x + y + z = 6$

C. $x + 2y + 4z = 8$

D. $x - 5y + 2z = -1$

E. $x - 2y + z = 1$

2. Compute the angle ω between $\mathbf{i}-\mathbf{j}+2\mathbf{k}$ and $-\mathbf{i}-\mathbf{j}+2\mathbf{k}$. Then $\omega =$

A. $\cos^{-1}\left(\frac{2}{3}\right)$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{2\pi}{3}$

E. $\frac{\pi}{6}$

3. The plane through the point $(1, 2, 3)$ parallel to the lines

$$x = 1 + 2t, \quad y = \pi + t, \quad z = 11,$$

and

$$x = \sqrt{2} + 4t, \quad y = 1/5 + t, \quad z = 1/7 - t$$

is given by the equation

A. $\sqrt{2}x - y + 2z = 4 - \sqrt{2}$

B. $2x - y + z = 3$

C. $x - y = -1$

D. $x - 2y + 2z = 3$

E. $11x + y - 3z = 4$

4. Determine the value of the parameter a so that the line

$$x = 4 + 5t, \quad y = 2 + t, \quad z = 7 + at$$

and the plane

$$x - 2y - z = 3$$

do not intersect.

A. 1

B. 3

C. 2

D. -1

E. 4

5. The surface $2x^2 - y^2 + z^2 = 1$ looks most like

6. The linear approximation of $f(x, y) = x\sqrt{y}$ at $(1, 4)$ is

A. $2 + 2x - y/4$

B. $2 + 2x + y/4$

C. $2 - 2x + y/4$

D. $2x + y/4 - 1$

E. $2x - y/4 - 1$

7. Find $\iint_S \mathbf{F}\cdot d\mathbf{S}$, where $\mathbf{F} = (x, y, z)$, S is the portion of the paraboloid $z = x^2 + y^2$ below the plane $z = 4$, and the orientation of S is chosen so that $\mathbf{n}$ is the upward normal.

A. 16π

B. -8π

C. $-\pi/2$

D. $\pi/2$

E. 24π

8. Let z be a differentiable function of x, y satisfying

$$e^{xz}\sin y = yz.$$

The tangent plane at $(0, \pi/2, 2/\pi)$ is

A. $-x + 4y + \pi^2 z = 1$

B. $-4x + 4y + \pi^2 z = -1$

C. $4x - 4y - \pi^2 z = 1$

D. $-4x + 4y + \pi^2 z = 4\pi$

E. $-x + y + 4z = \pi$

9. Let $z = f(x, y)$ is differentiable, and let $x = s^2 - t$, $y = t^3 \ln(1 + s)$. Then $\partial z/\partial s$ at $s = 0$, $t = 0$ is

A. 1

B. 0

C. -1

D. cannot be determined

E. 1/2

10. The tangent plane to the surface $z = xy + x + y$ at $(0, 0, 0)$ intersects the xz-plane in the line

A. $z = x$

B. $z = x + 1$

C. $z = -x$

D. $z = -x - 1$

E. $2x$

11. Find the total flux of the vector field

$$\mathbf{F} = (3x, xy, 1)$$

across the boundary of the box $D = \{|x| \leq 1,\ |y| \leq 2,\ |z| \leq 3\}$.

A. 144

B. 0

C. 72

D. 9

E. 27

12. The surface S is best represented parametrically by

A. $\mathbf{r}(u,v) = (3\cos u, 3\sin u, v)$, $-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$, $1 \leq v \leq 4$

B. $\mathbf{r}(u,v) = (3\sin u, 3\cos u, v)$, $-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$, $1 \leq v \leq 4$

C. $\mathbf{r}(u,v) = (3\sin u, v, 3\cos u)$, $0 \leq u \leq \pi$, $1 \leq v \leq 4$

D. $\mathbf{r}(u,v) = (3\cos u, v, 3\sin u)$, $-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$, $1 \leq v \leq 4$

E. $\mathbf{r}(u,v) = (3\sin u, v, 3\cos u)$, $-\frac{\pi}{2} \leq u \leq \frac{\pi}{2}$, $1 \leq v \leq 4$

13. Let R be bounded by the curve $y = 2 - x^2$ and the line $y = x$, and let C be its boundary, positively oriented. Then

$$\int_C (x^2 + y)dx + (3x + y^2)dy =$$

A. 9

B. $\frac{10}{3}$

C. $\frac{1}{3}$

D. $\frac{7}{2}$

E. $\frac{4}{3}$

14. The surface

$$x = 2\cos u \sin v, \quad y = 3\sin u \sin v, \quad z = 10\cos v,$$

$0 \leq u \leq \pi/2$, $\pi/4 \leq v \leq \pi/2$, looks most like

15. Let f be a scalar function and $\mathbf{F}$ a vector field in $\mathbf{R}^3$. Which of the following expressions are meaningful

(I) $\operatorname{grad} \mathbf{F}$
(II) $(\operatorname{grad} f) \times \mathbf{F}$
(III) $\operatorname{div} f$
(IV) $\operatorname{curl} \mathbf{F}$
(V) $\operatorname{curl}(\operatorname{div} \mathbf{F})$

A. II, IV, V

B. I, II, IV

C. I, IV

D. I, II, IV

E. II, IV

16. Use Green's theorem to compute the area of the region D bounded by the x-axis and the arch of the cycloid

$$x = t - \sin t, \quad y = 1 - \cos t, \quad 0 \le t \le 2\pi$$

A. 2π

B. 3π

C. $\pi/2$

D. $2\pi/3$

E. π

17. Let C be the curve $x = e^y$ from $(1, 0)$ to $(e, 1)$. Then

$$\int_C xe^y \, dx =$$

A. $e^3 - 1$

B. $e^3/3 - 1/3$

C. $e^3/3$

D. $e^2 - 1$

E. e^2

18. The area of the portion of the plane $x+3y+2z = 6$ in the first octant is

A. $3\sqrt{11}$

B. $3\sqrt{14}$

C. $6\sqrt{14}$

D. $6\sqrt{7}$

E. $6\sqrt{11}$

19. Which of the following statements is true about

$$f(x,y) = \frac{1}{3}x^3 + yx - \frac{1}{3}y^3$$

at $(1,-1)$:

A. $f(1,-1)$ is a local min.

B. $f(1,-1)$ is a local max.

C. $(1,-1)$ is a saddle point

D. $z = x - y - \frac{7}{3}$ is the tangent plane at $(1,-1)$

E. $\nabla f(1,-1) = 2\mathbf{i} - 2\mathbf{j}$

20. The region of integration of the iterated integral

$$\int_0^{\pi/4} \int_0^{3\sec\theta} r\, dr\, d\theta$$

is

A. a rectangle

B. one loop of a rose curve

C. a cardioid

D. a circular sector

E. a triangle

21. If

$$\int_0^1 \int_{x^3}^1 f(x,y)\,dy\,dx = \int_0^1 \int_a^b f(x,y)\,dx\,dy,$$

then $(a,b) =$

A. $(y^{1/3}, 1)$

B. $(0, y^{1/3})$

C. $(y^{1/3}, 0)$

D. $(1, x^3)$

E. $(1, y^3)$

22. For the curve

$$x = a\cos(2t), \quad y = a\sin(2t), \quad z = 3tb,$$

the parameter t is an arc length parameter, if

A. $a^2 + b^2 = 13$

B. $4a^2 + 9b^2 = 1$

C. $2a^2 + 3b^2 = 1$

D. $3a^2 + 2b^2 = 1$

E. $4a^2 + 9b^2 = 13$