MATH 261 - SPRING 2002

FINAL EXAM

Name ______________________ Instructor ______________________

Signature ______________________ Recitation Instructor ______________________

Div. Sect. No. ______________________

FINAL EXAM INSTRUCTIONS

1. You must use a #2 pencil on the mark-sense sheet (answer sheet).
2. On the mark-sense sheet, fill in the instructor's name and the course number.
3. Fill in your name and student identification number and blacken in the appropriate spaces.
4. Mark in your division and section number of your class. For example, for division 02, section 03, fill in 0203 and blacken the corresponding circles, including the circles for the zeros. (If you do not know your division and section number ask your instructor.)
5. Sign the mark-sense sheet.
6. Fill in the information above and fill in your name on each of the question sheets.
7. There are 20 questions, each worth 10 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–20. Do all your work on the question sheets. Turn in both the mark-sense sheets and the question sheets when you are finished.
8. You must show sufficient work to justify all answers. Correct answers with no work or inconsistent work may not be given credit.
9. Calculators are not allowed. NO BOOKS OR PAPERS ARE ALLOWED. Use the back of the test pages for scratch paper.

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1. Find the vector projection of $\vec{b} = \vec{i} - 6\vec{j} + 5\vec{k}$ onto $\vec{a} = \vec{i} - 2\vec{j} + \vec{k}$.

A. $\frac{9}{31}(\vec{i} - 6\vec{j} + 5\vec{k})$

B. $18\vec{i} - 36\vec{j} + 18\vec{k}$

C. $18\vec{i} - 108\vec{j} + 90\vec{k}$

D. $3\vec{i} - 6\vec{j} + 3\vec{k}$

E. $10\vec{i} + 4\vec{j} + 4\vec{k}$

2. Find a unit vector orthogonal to the plane having parametrization
$\vec{r}(s,t) = (3 + s + t)\vec{i} + s\vec{j} + (5 + s + 2t)\vec{k}$

A. $\frac{1}{\sqrt{6}}(2\vec{i} - \vec{j} - \vec{k})$

B. $\frac{11}{\sqrt{3}}(\vec{i} + \vec{j} + \vec{k})$

C. $\frac{1}{\sqrt{2}}(\vec{i} + \vec{k})$

D. $\frac{11}{\sqrt{6}}(\vec{i} + 2\vec{j} - \vec{k})$

E. $\frac{1}{\sqrt{3}}(\vec{i} - \vec{j} - \vec{k})$

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3. Find an equation in spherical coordinates for the surface with a cartesian equation $x^2 + y^2 - z^2 = 1$.

A. $\rho^2(\cos^2\phi - \sin^2\phi) = 1$

B. $\rho^2(\sin^2\phi - \cos^2\phi) = 1$

C. $\rho^2(\cos^2\theta - \sin^2\theta)\cos^2\phi = 1$

D. $\rho^2\cos\theta\sin\phi = 1$

E. $\rho^2 - \rho\sin\phi = 1$

4. Find the length of the curve $\vec{r}(t) = \frac{2}{3}(1+\sin t)^{\frac{3}{2}}\vec{i} + \frac{2}{3}(1-\sin t)^{\frac{3}{2}}\vec{j} + \sqrt{2}\cos t\vec{k}, \quad 0 \le t \le \pi$.

A. 2π

B. $\frac{\pi}{2}$

C. $\sqrt{2}\pi$

D. $\frac{\pi}{\sqrt{2}}$

E. π

5. $\lim\limits_{(x,y)\to(0,0)} \dfrac{(x-y)^2}{x^2+y^2}$

A. $= 1$

B. $= 0$

C. $= +\infty$

D. $= 2$

E. Does not exist

6. Find an equation of the tangent plane to the surface $f(x,y) = \ln(3x - y)$ at the point $(1, 2, 0)$.

A. $3x - y - z = 1$

B. $3x + y - z = 5$

C. $x + 3y - z = 7$

D. $3x - y = 1$

E. $x + y - z = 3$

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7. If f is a differentiable function of one variable and $z = u + f(u^2v^2)$, then $u\,\dfrac{\partial z}{\partial u} - v\,\dfrac{\partial z}{\partial v}$ equals

A. 1

B. u

C. v

D. uv

E. 0

8. Find the directional derivative of $f(x,y) = x^2y^2$ at the point $(1,1)$ in the direction of $\vec{i} + 2\vec{j}$.

A. $\dfrac{6}{\sqrt{5}}$

B. 1

C. 2

D. $\dfrac{\sqrt{5}}{12}$

E. $\dfrac{\sqrt{5}}{12}$

9. Let $f(x, y) = -x^2 + 6x - y^2 + 4y - 2$. Which of the following is true?

A. The minimum value of f is -2

B. The maximum value of f is -2

C. The minimum value of f is 11

D. The maximum value of f is 11

E. f has a saddle point

10. Find the maximum value of $f(x, y, z) = xyz$ in the first octant subject to the constraint $2x + 2y + z = 6$.

A. $\frac{5}{2}$

B. $\frac{3}{2}$

C. 3

D. 4

E. 2

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11. Compute the area of the region bounded by the parabolas $x = y^2$ and $x = 4y^2 - 3$.

A. 8

B. 3

C. 4

D. 5

E. 6

12. Convert the iterated integral $\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dydx$ to an iterated integral in polar coordinates.

A. $\int_0^{\frac{\pi}{2}} \int_0^{2\cos\theta} r^2 dr d\theta$

B. $\int_0^{\pi} \int_0^{2\cos\theta} r^2 dr d\theta$

C. $\int_0^{\pi} \int_0^{2\cos\theta} r^2 dr d\theta$

D. $\int_0^{\pi} \int_0^{2\sin\theta} r\, dr d\theta$

E. $\int_0^{\frac{\pi}{2}} \int_0^{1} r^2 dr d\theta$

13. Find the surface area of that part of the parabolic cylinder $z = x^2$ that lies above the triangle with vertices $(0,0,0)$, $(1,0,0)$ and $(1,1,0)$.

A. $\dfrac{5^{\frac{3}{2}} - 1}{12}$

B. $\dfrac{6^{\frac{3}{2}} - 1}{12}$

C. $\dfrac{17^{\frac{3}{2}} - 1}{12}$

D. $\dfrac{3^{\frac{3}{2}} - 1}{12}$

E. $\dfrac{7^{\frac{3}{2}} - 3^{\frac{3}{2}}}{12}$

14. An object occupies the region in space bounded above by the sphere $x^2 + y^2 + z^2 = 36$ and below by the upper nappe of the cone $3z^2 = x^2 + y^2$. The mass density of the object is its distance from the xy plane. Set up a triple integral for the total mass m of the object in spherical coordinates.

A. $\displaystyle\int_0^{2\pi} \int_0^{\frac{\pi}{6}} \int_0^6 \rho^3 \cos\phi \sin\phi \, d\rho d\phi d\theta$

B. $\displaystyle\int_0^{2\pi} \int_0^{\frac{\pi}{6}} \int_0^6 \rho^3 \sin^2\phi \, d\rho d\phi d\theta$

C. $\displaystyle\int_0^{2\pi} \int_0^{\frac{\pi}{3}} \int_0^6 \rho^3 \cos\phi \sin\phi \, d\rho d\phi d\theta$

D. $\displaystyle\int_0^{2\pi} \int_0^{\frac{\pi}{3}} \int_0^6 \rho^3 \sin^2\phi \, d\rho d\phi d\theta$

E. $\displaystyle\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^6 \rho^3 \cos\phi \sin\phi \, d\rho d\phi d\theta$

15. The line integral $\int_C -zdx + xdz$ on the curve C given by $\overrightarrow{r(t)} = \frac{1}{2}(\cos t)\vec{i} + \frac{1}{2}(\sin t)\vec{k}$ for $0 \le t \le \pi$ is:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{2}$

E. π

16. Are the following statements true or false?

1. The line integral $\int_C (e^x + z)dx + (x + 2z)dz$ is independent of path.
2. $\int_C (e^x + z)dx + (x + 2z)dz = 0$ for every smooth closed curve in space.
3. $\vec{F}(x, y, z) = (e^x + z)\vec{i} + (x + 2z)\vec{k}$ is not conservative.

A. All 3 are false

B. 1) and 2) are true, 3) is false

C. 1) and 2) are false, 3) is true

D. 1) is true, 2) and 3) are false

E. All 3 are true.

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17. Let C be the closed curve formed by traversing the boundary of the region bounded by $y = 1 - x^2$ and $y = 0$ in a counterclockwise direction. By Green's Theorem, $\int_C y^2 dx + 3y^2x^2 dy$ equals

A. $\int_{-1}^{1} \int_{0}^{1-x^2} (6xy^2 - 2y) dy dx$

B. $\int_{-1}^{1} \int_{0}^{1-x^2} (2y - 6xy^2) dy dx$

C. $\int_{-1}^{1} \int_{0}^{1-x^2} (6xy^2 + 2y) dy dx$

D. $\int_{-1}^{1} \int_{0}^{1-x^2} 6yx^2 dy dx$

E. $\int_{-1}^{1} \int_{0}^{1-x^2} (-6yx^2) dy dx$

18. If Σ is the surface formed by the part of $z = 9 - x^2 - y^2$ above the xy plane, $\vec{n}$ is directed upward, and $\vec{F}(x, y, z) = y\vec{i} - x\vec{j} + z\vec{k}$, $\iint_\Sigma \vec{F} \cdot \vec{n} dS$ equals:

A. 0

B. $\frac{81\pi}{2}$

C. 81π

D. $\frac{17\pi}{2}$

E. 17π

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19. Let Σ be the sphere $x^2 + y^2 + z^2 = 4$ and let $\vec{F}(x, y, z) = 3x\vec{i} + 4y\vec{j} + 5z\vec{k}$. Use the divergence theorem to evaluate $\iint_\Sigma \vec{F} \cdot \vec{n}\, dS$ where $\vec{n}$ is the outward unit normal.

A. 0

B. 4π

C. 16π

D. 64π

E. 128π

20. Let S be the surface formed by the part of $z = 9 - x^2 - y^2$ above the xy plane, and let $\vec{n}$ be the upward unit normal on S. Let C be the curve $x^2 + y^2 = 9$, $z = 0$, oriented counterclockwise, and assume $\vec{F}(x, y, z) = xz\vec{i} + y^2\vec{j} + x^2\vec{k}$. Which of the following is Stokes' Theorem for $\vec{F}$ on S?

A. $\iint_S (xz\vec{i} + y^2\vec{j} + x^2\vec{k}) \cdot \vec{n}\, dS = \int_C xzdx + y^2dy$

B. $\iint_S (xz\vec{i} + y^2\vec{j} + x^2\vec{k}) \cdot \vec{n}\, dS = \int_C -y^2dx + xzdx$

C. $\iint_S (xz\vec{i} + y^2\vec{j} + x^2\vec{k}) \cdot \vec{n}\, dS = 0$

D. $\iint_S -x\vec{j} \cdot \vec{n}\, dS = \int_C xzdx + y^2dy$

E. $\iint_S x\vec{j} \cdot \vec{n}\, dS = 0$