NAME ____________________

STUDENT ID # ____________________

INSTRUCTOR ____________________

INSTRUCTIONS

1. There are 14 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. I.D.# is your 9 digit ID (probably your social security number). Also write your name at the top of pages 2–14.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet for each question.
4. No books, notes or calculators may be used on this exam.
5. Each problem is worth 8 points. The maximum possible score is 200 points.
6. Using a #2 pencil, fill in each of the following items on your answer sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION, write in your division and section number and fill in the little circles. (For example, for division 9 section 1, write 0901. For example, for division 38 section 2, write 3802).
   (c) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your student ID number, and fill in the little circles.
   (d) Using a #2 pencil, put your answers to questions 1–25 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
   (e) Sign your answer sheet.
7. After you have finished the exam, hand in your answer sheet and your test booklet to your instructor.

1. Find an equation of the plane that contains the point $(1, 2, -3)$ and the line with symmetric equations $x - 2 = y - 1 = \dfrac{z+2}{2}$.

A. $x + y + 5z = -12$

B. $2x + y + 5z = -11$

C. $5x + 5y + z = 20$

D. $3x + y - 2z = 11$

E. $x + 2y - 3z = 14$

2. Find the length of the curve given by $\vec{r}(t) = (t+2)\vec{i} - (t+1)\vec{j} + 2t\vec{k},\ 0 \le t \le 3$.

A. 12

B. 6

C. $3\sqrt{6}$

D. $3\sqrt{3}$

E. 9

3. A particle is moving with acceleration $t\vec{j} + \vec{k}$. If the velocity at time $t = 1$ is $\vec{v}(1) = \vec{i} - \frac{1}{2}\vec{j}$, then what is the velocity at time $t = 0$?

A. $\vec{i} - \vec{j} - \vec{k}$

B. $\vec{i} - \vec{j} + 2\vec{k}$

C. $\vec{i} - \frac{1}{2}\vec{j} - \vec{k}$

D. $\vec{i} + \frac{1}{2}\vec{j} - \vec{k}$

E. $-\vec{i} + \vec{j}$

4. The tangent plane to the surface $x^2 + y^2 - z^2 = 4$ at the point $(1, 2, -1)$ is given by:

A. $x + 2y + z = 4$

B. $2x + 4y + z = 9$

C. $2x + 4y - z = 11$

D. $x + 2y - z = 6$

E. $x + y - z = 4$

5. If C is a curve defined by $r(t) = (2t + 2,\ -t^2 - t)$, then a unit normal vector to C at $(4, -2)$ is given by

A. $\frac{1}{\sqrt{5}}\,\vec{i} + \frac{2}{\sqrt{5}}\,\vec{j}$

B. $\frac{2}{\sqrt{13}}\,\vec{i} - \frac{3}{\sqrt{13}}\,\vec{j}$

C. $\frac{2}{\sqrt{5}}\,\vec{i} + \frac{1}{\sqrt{5}}\,\vec{j}$

D. $\frac{3}{\sqrt{13}}\,\vec{i} + \frac{2}{\sqrt{13}}\,\vec{j}$

E. $2\vec{i} + 4\vec{j}$

6. The surface defined by the equation $z = 1 + \sqrt{1 + x^2 + y^2}$ is:

A. an ellipse

B. a cylinder

C. an ellipsoid

D. a paraboloid

E. a hyperboloid

7. Suppose $w = f(x, y)$ satisfies $\dfrac{\partial f}{\partial x}(2,3) = 3$ and $\dfrac{\partial f}{\partial y}(2,3) = -1$. If $x(t) = t^3 + 1$ and $y(t) = 5t^2 - 2$, then compute $\dfrac{dw}{dt}(1)$.

A. 2

B. 3

C. 1

D. -1

E. 13

8. Find the directional derivative of $f(x, y, z) = x^2 - xz + y^2$ at the point (1,2,0) in the direction $2\vec{i} + \vec{j} - 2\vec{k}$.

A. $\dfrac{2}{3}$

B. 2

C. $\dfrac{10}{3}$

D. $\dfrac{8}{3}$

E. 4

9. Which of the following is a smooth parameterization of the oriented curve traveling counterclockwise on the semi–circle $x^2 + y^2 = 1$ with $y \leq 0$ from the point $(-1, 0)$ to the point $(1, 0)$?

A. $\vec{r}(t) = \cos t\vec{i} - \sin t\vec{j},\ 0 \leq t \leq \pi$

B. $\vec{r}(t) = -\cos t\vec{i} - \sin t\vec{j},\ 0 \leq t \leq \pi$

C. $\vec{r}(t) = \cos 2t\vec{i} + \sin 2t\vec{j},\ 0 \leq t \leq \frac{\pi}{2}$

D. $\vec{r}(t) = \cos 2t\vec{i} - \sin 2t\vec{j},\ \dfrac{\pi}{2} \leq t \leq \pi$

E. None of these

10. The function $f(x, y) = \dfrac{1}{3}x^3 - 4xy + y^2 - 9x$ has:

A. a relative minimum and a relative maximum

B. two saddle points

C. two relative minima

D. a relative maximum and a saddle point

E. a relative minimum and a saddle point

11. Find the maximum value of $x + y$ along the curve defined by $x^2 + 2y^2 = 6$.

A. 3

B. $4\sqrt{3}$

C. $2\sqrt{6}$

D. 4

E. $2\sqrt{2}$

12. Let D be the portion of the disk $x^2 + y^2 < 4$ satisfying $y > \sqrt{3}$.
Compute $\iint_D y dA$.

A. $4 - 2\sqrt{3}$

B. $\frac{2}{3}$

C. $\frac{2\sqrt{3}}{3}$

D. $1 + \frac{\sqrt{3}}{2}$

E. $\frac{4}{3}$

13. If $f(x,y) = x\sin xy^2$, compute $\dfrac{\partial^2 f}{\partial x \partial y}$.

A. $2xy\cos xy^2 - 2x^2y^3\sin xy^2$

B. $2xy\cos xy^2 - 2xy\sin xy^2$

C. $4xy\cos xy^2 + 2x^2y^3\sin xy^2$

D. $2xy\cos xy^2 + 2x^2y^3\sin xy^2$

E. $4xy\cos xy^2 - 2x^2y^3\sin xy^2$

14. Evaluate the integral $\displaystyle\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} x^2 + y^2 dydx$ using polar coordinates.

A. $\dfrac{\pi}{6}$

B. $\dfrac{\pi}{2}$

C. π

D. $\dfrac{\pi}{4}$

E. $\dfrac{\pi}{3}$

15. Find the volume of the region bounded by the cylinder $x^2 + y^2 = 1$, the xy–plane and the surface $z = 1 + e^{-x^2-y^2}$.

A. $\pi\left(2 - \frac{1}{e}\right)$

B. $\pi\left(1 + \frac{1}{e}\right)$

C. $\frac{2\pi}{e}$

D. πe

E. $\pi(1 - e^{-2})$

16. Find the surface area of the portion of the plane $3x + \sqrt{6}y + z = 10$ that is directly above the triangle with vertices $(0,0,0)$, $(1,1,0)$, and $(0,1,0)$.

A. 1

B. $\frac{1}{2}\sqrt{5}$

C. 2

D. $\sqrt{11}$

E. $2\sqrt{13}$

17. An object occupies the tetrahedron in the first octant bounded by the coordinate planes and the plane $4x+2y+z=4$. The mass density at (x,y,z) equals the distance from that point to the z–axis. Which of the following integrals gives the total mass of the object?

A. $\int_0^1\int_0^2\int_0^4 (x^2+y^2)\,dzdydx$

B. $\int_0^1\int_0^2\int_0^{4-2y-4x} (x^2+y^2)^{\frac{1}{2}}\,dzdydx$

C. $\int_0^2\int_0^{\frac{2-y}{2}}\int_0^{4-2y-4x} (x^2+y^2)\,dzdxdy$

D. $\int_0^2\int_0^{\frac{2-y}{2}}\int_0^{4-2y-4x} (x^2+y^2)^{\frac{1}{2}}\,dzdxdy$

E. $\int_0^1\int_0^2\int_0^4 (x^2+y^2)^{\frac{1}{2}}\,dzdydx$

18. Find the volume of the region D inside the sphere $x^2+y^2+z^2=1$, below the cone $z=\sqrt{3(x^2+y^2)}$ and above the xy–plane.

A. $\dfrac{\pi}{\sqrt{6}}$

B. $\dfrac{\pi}{6}$

C. $\dfrac{\pi}{\sqrt{3}}$

D. $\dfrac{\pi}{2}$

E. $\dfrac{\sqrt{3}\pi}{4}$

19. Compute curl $\vec{F}$, where
$\vec{F}(x, y, z) = (x + y)\vec{i} + yz\vec{j} + \sin x\vec{k}$.

A. 0

B. $-y\vec{i} - \cos x\vec{j} - \vec{k}$

C. $-z\vec{i} + \cos x\vec{j}$

D. $-yz\vec{i} + \sin x\vec{j}$

E. $-y\vec{i} + \sin x\vec{j} + z\vec{k}$

20. Let $f(x, y)$ be a function such that grad $f(x, y) = (2xy + 1)\vec{i} + x^2\vec{j}$. If $f(1, 1) = 1$, then $f(1, 2)$ equals

A. 2

B. 1

C. 3

D. 4

E. 0

21. Let C be the counterclockwise curve given by the triangle with vertices $(0,0)$, $(1,0)$, and $(1,2)$. Compute $\int_C -xy^2dx + 3x^2ydy$.

A. 4
B. $\frac{7}{2}$
C. 3
D. $\frac{9}{2}$
E. $\frac{15}{4}$

22. Let C be the curve parametrized by $\vec{r}(t) = (2t+1)\vec{i} + 3t^2\vec{j}$, for $0 \leq t \leq 2$. If $\vec{F}(x,y) = -2y\vec{i} + (x+1)\vec{j}$, compute the line integral $\int_C \vec{F} \cdot d\vec{r}$.

A. -18
B. $\frac{15}{2}$
C. 24
D. 12
E. $\frac{21}{2}$

23. Let $\vec{F} = \text{grad } f$, where $f(x, y, z) = x^2 - y^2 z$, and let a curve C be defined by $\vec{r}(t) = (\cos \pi t)\vec{i} + (\sin \pi t)\vec{j} + 2t\vec{k}$ for $0 \leq t \leq \frac{1}{2}$. Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$.

A. $-\dfrac{3\pi}{2}$

B. 1

C. -2π

D. -2

E. $\dfrac{1}{2}$

24. Let Σ be the portion of the paraboloid $z = x^2 + y^2$ satisfying $z < 4$. Compute the surface integral $\int_\Sigma (1 + 4x^2 + 4y^2)^{-\frac{1}{2}}\, dS$.

A. 16π

B. 2π

C. $\pi(\sqrt{17} - 1)$

D. 4π

E. $2\pi\sqrt{17}$

25. Let D be the solid cylinder defined by $x^2 + y^2 < 4$ and $0 < z < 2$, and let $\sum$ be the boundary of D. If $\vec{n}$ is the outward pointing normal and $\vec{F}(x, y, z) = \frac{x^3}{3}\,\vec{i} + \frac{y^3}{3}\,\vec{j} + \vec{k}$, compute $\int_{\Sigma} \vec{F} \cdot \vec{n} dS$.

A. 16π

B. $\frac{32\pi}{3}$

C. $\frac{8\pi}{3}$

D. 8π

E. $\frac{16\pi}{3}$