Math 26100 Final

12/14/15

$\boxed{\textbf{Version01}}$

Name:

Section number:

10 digit PU-ID Number:

Signature:

(1) **Do Not Open** until instructed to do so.
(2) You have to be in your section and in your assigned seat.
(3) When time is called: **Remain in Your Seat.**
(4) You may not use any electronic devices or have them out.
(5) You may not have anything else out, like paper, extra pens etc. Just the exam, scantron, one pencil and eraser (optional).
(6) Use No 2 pencil.
(7) **Mark Your Test With The Version Number!**
(8) Sign the exam policies on the next page.
(9) There are 20 problems each worth 10 points.
(10) If you take the exam apart, mark each page with your name.

General Exam Policies

(1) Students may not open the exam until instructed to do so.
(2) Students must obey the orders and requests by all proctors, TAs, and lecturers.
(3) No student may leave in the first 20 min or in the last 10 min of the exam.
(4) Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
(5) After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
(6) Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

PROBLEM 1: Find the area of the triangle that has vertices at $P(1,2,1)$, $Q(2,3,2)$, and $R(0,2,3)$.

A. $\sqrt{3}$

B. $\sqrt{2}$

C. $\sqrt{14}$

D. $\sqrt{14}/2$

E. $\sqrt{17}/2$

PROBLEM 2: Find $\mathbf{r}(1)$ if $\mathbf{r}''(t) = 12t\mathbf{i} + 12t^2\mathbf{j} + \mathbf{k}$ and $\mathbf{r}(0) = \mathbf{j}$ and $\mathbf{r}'(0) = -\mathbf{k}$.

A. $2\mathbf{i} + 2\mathbf{j} - \frac{1}{2}\mathbf{k}$

B. $6\mathbf{i} - \mathbf{j}$

C. $2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$

D. $2\mathbf{i} + \mathbf{k}$

E. $2\mathbf{i} + \mathbf{j} + \frac{1}{2}\mathbf{k}$

PROBLEM 3: Find the length of the curve $\mathbf{r}(t) = \langle 2t, t^2, \frac{1}{3}t^3 \rangle$, $0 \le t \le 1$.

A. 7/2

B. 13/2

C. 9/2

D. 20/3

E. 7/3

PROBLEM 4: If $f(x, y) = \frac{x^2y - y^2}{2x + 3y}$, compute $\frac{\partial f}{\partial x}(1, 1)$.

A. $\frac{4}{25}$

B. $\frac{1}{25}$

C. $-\frac{4}{25}$

D. $\frac{2}{5}$

E. $-\frac{2}{5}$

PROBLEM 5: If z is given implicitly as a function of x and y by the equation $e^z - xy^2z - 2 = 0$, then $\frac{\partial z}{\partial y}$ is:

A. $\dfrac{2xyz}{e^z}$

B. $\dfrac{2xyz}{e^z - xy^2}$

C. $\dfrac{e^z - 2xyz}{xy^2}$

D. $\dfrac{1}{e^z - xy^2}$

E. $\dfrac{e^z}{xy^2}$

PROBLEM 6: Use a linear approximation of $f(x, y) = 3(y + 2x^2)^{\frac{1}{2}}$ to compute an approximate value of $f(2.1, 0.9)$.

A. 8.55

B. 9.45

C. 9.35

D. 8.65

E. 9.1

PROBLEM 7: Assume $z = x^2 + (1 + \sin y)^5$, $x = st^2$, and $y = 2st$. What is $\frac{\partial z}{\partial s}$ when $s = \frac{1}{2}$ and $t = \pi$?

A. $\pi^4 + 120\pi$

B. $\pi^2 + 10\pi$

C. $\pi^4 - 10\pi$

D. $\pi^4 + 10\pi$

E. $\pi^4 - 120\pi$

PROBLEM 8: A vector that is perpendicular to the curve $4 \ln x - e^y = 4 \ln 2 - 5$ at the point $(\frac{2}{e}, 0)$ is

A. $< \frac{1}{2e}, 1 >$

B. $< \frac{1}{e}, -1 >$

C. $< 2e, -1 >$

D. $< 2e, 1 >$

E. $< -1, 2e >$

PROBLEM 9: The function $f(x, y) = x^3 + y^3 - 3xy + 4$ has

A. 2 local maxima

B. 2 local minima

C. 2 saddle points

D. 1 local minimum and 1 saddle point

E. 1 local maximum and 1 saddle point

PROBLEM 10: Find the absolute max and min values of $f(x, y) = \sqrt{2}x + \sqrt{2}y - x^2 - y^2$ on the disk $x^2 + y^2 \leq 1$.

A. 1 and -3

B. 0 and -1

C. 4 and -4

D. 2 and -2

E. 1 and 0

PROBLEM 11: Calculate the integral

$$\int_0^{12} \int_0^{\sqrt{\frac{x}{3}}} \sqrt{12y - y^3}\, dydx$$

by reversing the order of integration.

A. 0

B. $\frac{32}{3}$

C. $\frac{64}{9}$

D. $\frac{128}{9}$

E. $\frac{128}{3}$

PROBLEM 12: Let D be the part of the disc $x^2 + y^2 \leq 1$ inside the upper half plane given by $y \geq 0$

Calculate

$$\iint_D y^2 dA$$

A. $\frac{\pi}{4}$

B. $\frac{\pi}{8}$

C. 2π

D. 0

E. 4π

PROBLEM 13: Evaluate the triple integral

$$\iiint_E z\, dV$$

where $E = \{(x, y, z)|0 \leq x \leq 1 - y - z,\ 0 \leq y \leq 1 - z,\ 0 \leq z \leq 1\}$.

A. $\frac{1}{24}$

B. $\frac{1}{12}$

C. 1

D. 2

E. $\frac{1}{2}$

PROBLEM 14: Let

$$\mathbf{F} = \langle y, x + e^z, ye^z + 1\rangle$$

Calculate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

where C is the curve $r(t) = \langle t^2, t^3, \sin(t\pi)\rangle : 0 \leq t \leq 2$

A. -1

B. 0

C. 2

D. 16

E. 40

PROBLEM 15: Let $C := C_1 \cup C_2$ be the curve which is composed of C_1 and C_2, where C_1 is the circle centered at $(0, 0)$ of radius 1 oriented clockwise and C_2 is the circle centered at $(0, 0)$ of radius 2 oriented counterclockwise. Consider the vector field

$$F = \langle x^2 + y^2, x \rangle$$

Compute the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}$$

A. 3π

B. -3π

C. 0

D. 5π

E -5π

PROBLEM 16: Let S be the piece of the surface $z = xy + 5$ that lies inside the cylinder $x^2 + y^2 = 4$ with upward normal. Calculate

$$\iint_S 2\, dS$$

A. $\frac{4\pi}{3}(5\sqrt{5} - 1)$

B. $\frac{2\pi}{3}(5\sqrt{5} - 1)$

C. $\frac{8\pi}{3}(5\sqrt{5} - 1)$

D. $\frac{4\pi}{3}$

E. $\frac{2\pi}{3}(\sqrt{5} - 1)$

Problem 17: Let S be the part of paraboloid $z = x^2 + y^2$ which is below $z = 4$ with downward normal. Let $\mathbf{F}$ be the vector field $\langle y, x, z \rangle$. Compute the flux

$$\iint_S \mathbf{F} \cdot d\mathbf{S}$$

A. -16π

B. -8π

C. 0

D. 8π

E. 16π

Problem 18: Consider the vector field

$$\mathbf{F}(x, y, z) = \langle x^3 + xy^2 + xz^2, x^2y + y^3 + yz^2, x^2z + y^2z + z^3 \rangle$$

and let S be the sphere of radius 2 centered at the origin with positive orientation, i.e. outward pointing normal. Compute the flux

$$\iint_S \mathbf{F} \cdot d\mathbf{S}$$

A. -4π

B. -2π

C. $\frac{4}{3}\pi$

D. 64π

E. 128π

PROBLEM 19: Which of the following is true for the vector field

$$\mathbf{F} = \frac{1}{|\mathbf{x}|^3}\mathbf{x}$$

(1) It is defined on all of $\mathbb{R}^3$.
(2) The vector field satisfies $div(\mathbf{F}) = 0$ on its domain of definition.
(3) $\iint_S \mathbf{F} \cdot d\mathbf{S} = 0$ for any closed surface S which lies in the domain of definition of $\mathbf{F}$.
(4) The vector field is conservative on its domain of definition.
(5) $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for all closed curves C which lie in the domain of definition of $\mathbf{F}$.

A. 1, 3, 5

B. 2, 3

C. 2, 4, 5

D. 4, 5

E. All of the above

PROBLEM 20: Let $\mathbf{F}$ be a vector field whose components have continuous partial derivatives up to second order and f a function with continuous partial derivatives up to second order. Which of the following is true?

(1) $curl(grad(f)) = \mathbf{0}$
(2) $grad(div(\mathbf{F})) = \mathbf{0}$
(3) $div(curl(\mathbf{F})) = 0$
(4) $curl(curl(\mathbf{F}))$ is a vector field.
(5) $curl(div(\mathbf{F}))$ is a function.

A. 1, 3, 4

B. 1, 3

C. All of the above

D. 1, 2, 3, 4

E. 1, 3, 5

Extra Page

Extra Page