MA261 — FINAL EXAM — FALL 2013 – EXAM TYPE I

INSTRUCTIONS

1. **Fill in the information at the bottom of this page, but do not open this exam booklet until the class is told to begin the exam.**
2. Make sure the color of your scantron matches the color of the cover page of your exam.
3. Use a # 2 pencil to fill in the required information on your scantron (your name,student id, section number, etc.) and fill in the circles.
4. There are 12 different test pages (including this cover page). **Once you are allowed to open the booklet,** make sure you have a complete test.
5. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet.
6. Use a # 2 pencil to fill in the answers on your scantron.
7. The number of points each problem is worth is stated next to it. The maximum possible score is 200 points. No partial credit.
8. After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

RULES REGARDING ACADEMIC DISHONESTY

1) Students taking this exam will obey item 1 above. They are not allowed to seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only your instructor.
2) You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3) You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic device until after you have finished your exam, handed it in to your instructor and left the room.
4) Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course.
5) All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty

STUDENT NAME: ____________________

STUDENT SIGNATURE: ____________________

STUDENT ID NUMBER: ____________________

SECTION NUMBER: ____________________

1.**(9 points)** Evaluate $|\nabla f(1,0,3)|$, where $f(x,y,z) = xe^{yz} + z^2 \sin(y)$

A. 15

B. $\sqrt{135}$

C. $\sqrt{110}$

D. 10

E. $\sqrt{145}$

2. **(9 points)** Compute $\frac{\partial z}{\partial t}$, at $t = s = 1$ where $z = x^2 + y^2, x = 2t + s$, and $y = t + 2s$.

A. 24

B. 18

C. 32

D. 27

E. 12

3.(**9 points**) Find the critical point of the function $f(x, y) = x^3 - x^2y + y^2$ in the region $x > 0$, and $y > 0$.

A. $(2, \frac{3}{2})$

B. $(\frac{1}{2}, 4)$

C. $(\frac{1}{8}, 16)$

D. $(\frac{1}{6}, 12)$

E. $(3, \frac{9}{2})$

4.(**9 points**) The function $f(x, y) = 4x^3 + 12xy + 4y^3$ has two critical points: $(0, 0)$ and $(-1, -1)$. Which of the following is true?

A. $(-1, -1)$ corresponds to a local minimum and $(0, 0)$ to a saddle point.

B. $(-1, -1)$ corresponds to a local maximum and $(0, 0)$ to a saddle point.

C. $(-1, -1)$ corresponds to a saddle point and $(0, 0)$ to a local minimum.

D. Both $(-1, -1)$ and $(0, 0)$ correspond to saddle points.

E. $(-1, -1)$ corresponds to a local minimum and $(0, 0)$ corresponds to a local maximum

5.(**9 points**) The maximum of the function $f(x, y, z) = x+2y+4z$ on the sphere $x^2+y^2+z^2 = 21$ is equal to

A. $\frac{35}{4}$

B. $\frac{21}{4}$

C. 21

D. $\frac{21}{2}$

E. 35

6.(**9 points**) The directional derivative of the function $f(x, y, z) = 3x^2yz$ at the point $(1, 1, 1)$ in the direction of the vector $\vec{i} + 2\vec{j} + 2\vec{k}$ is

A. 6

B. $\frac{21}{4}$

C. 18

D. 12

E. 9

7.(**9 points**) The integral $\int\int_D (x+2y)\, dxdy$ where D is the region in $x \geq 0$ bounded by $y = x$, $y = -x$ and inside the circle $x^2 + y^2 = 1$

A. $\dfrac{\sqrt{3}}{3}$

B. $\dfrac{\sqrt{3}}{2}$

C. $\sqrt{2}$

D. $\dfrac{2\sqrt{2}}{3}$

E. $\dfrac{\sqrt{2}}{3}$

8.(**9 points**) A lamina bounded by $x = 0$, $y = x^2$ and $y = 1$ has density of mass $\rho(x, y) = x$. Its mass is equal to $\frac{1}{4}$ units. The x-coordinate of its center of mass $\bar{x}$ is equal to

A. $\dfrac{8}{15}$

B. $\dfrac{1}{5}$

C. $\dfrac{4}{5}$

D. $\dfrac{3}{10}$

E. $\dfrac{9}{10}$

9.(**9 points**) Find the volume of the solid region in the first octant bounded by the plane $x + y + z = 1$.

A. $\frac{1}{6}$

B. $\frac{1}{5}$

C. $\frac{1}{4}$

D. $\frac{1}{3}$

E. $\frac{1}{2}$

10.(**9 points**) Find the volume of the solid region bounded from above by $z = 1 - x^2 - y^2$ and from below by $z = x^2 + y^2 - 1$.

A. $\frac{2\pi}{3}$

B. $\frac{4\pi}{3}$

C. $\frac{8\pi}{3}$

D. π

E. 2π

11.(**9 points**) The volume of the region inside the sphere $x^2 + y^2 + z^2 = 4$ and the cone $z = \sqrt{3x^2 + 3y^2}$ is given by $\int_0^{2\pi} \int_0^{a} \int_0^{2} b \, d\rho \, d\phi \, d\theta$, where the values of a and b are

A. $a = \frac{\pi}{3}$, $b = \rho^2 \sin\phi$

B. $a = \frac{\pi}{6}$, $b = \rho^2 \sin\phi$

C. $a = \frac{\pi}{6}$, $b = \rho^2 \cos\phi$

D. $a = \frac{\pi}{3}$, $b = \rho^2 \cos\phi$

E. $a = \frac{\pi}{4}$, $b = \rho^2 \cos\phi$

12.(**9 points**) The length of the curve $x = 1 - 2t^2$, $y = 4t$, $z = 3 + 2t^2$, $0 \le t \le 2$ is given by

A. $\int_0^2 4\sqrt{2t^2 + 1}dt$

B. $\int_0^2 4\sqrt{t^2 + 5}dt$

C. $\int_0^2 \sqrt{8t + 4}dt$

D. $\int_0^2 \sqrt{8t^4 + 4t^2}\, dt$

E. $\int_0^2 4\, dt$

13.(**9 points**) Evaluate $\int_C (y-1)dx + x^2 dy$, where C is the line segment from $(0,1)$ to $(2,3)$.

A. $\frac{23}{6}$

B. $\frac{14}{3}$

C. $\frac{5}{6}$

D. $\frac{19}{6}$

E. 2

14.(**9 points**) Let E be the solid bounded above by $z = 1 - \frac{1}{2}(x^2 + y^2)$ and bounded below by $z = \frac{1}{2}(x^2 + y^2)$. Find the area of the surface of the boundary of E. Notice that there are two surfaces, and you need to find the sum of their areas.

A. $2\pi(2^{3/2} - 1)$

B. $\frac{2\pi}{3}(2^{3/2} - 1)$

C. $4\pi(2^{3/2} - 1)$

D. $\pi(2^{3/2} - 1)$

E. $\frac{4\pi}{3}(2^{3/2} - 1)$

15.(**9 points**) Let E be the solid bounded above by $z = 1 - \frac{1}{2}(x^2 + y^2)$, bounded below by $z = \frac{1}{2}(x^2 + y^2)$, with surface S, and outward pointing unit normal $\mathbf{n}$. Use the divergence theorem to find

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dS \text{ where } \mathbf{F}(x, y, z) = \langle x + y^3, x^2 - y, xy + z \rangle.$$

A. 2π

B. $\frac{4\pi}{3}$

C. $\frac{3\pi}{2}$

D. $\frac{\pi}{2}$

E. $\frac{\pi}{3}$

16.(**10 points**) Let Σ be the part of the surface $z = 1 - x^2 - y^2$ above $z = 0$, oriented by its upward pointing normal $\mathbf{n}$. Use Stokes' theorem to find

$$\iint_\Sigma \text{curl}\, \mathbf{F} \cdot \mathbf{n}\, dS \text{ where } \mathbf{F}(x, y, z) = \langle -y, zx, e^{xy} \rangle.$$

A. 3π

B. 2π

C. π

D. $-\pi$

E. -2π

17.(**9 points**). Let Σ be the graph of $z = xy$ for $0 \leq x \leq 2$, $0 \leq y \leq 2$, oriented with an upward pointing normal $\mathbf{n}$. Find $\iint_{\Sigma} \mathbf{F} \cdot \mathbf{n}\, dS$ where $\mathbf{F} = \langle x, -y, z \rangle$.

A. 1

B. 2

C. 4

D. 6

E. 8

18.(**9 points**) A potential for the vector field $\mathbf{F}(x, y) = \langle 3x^2y + 2xy,\ x^3 + x^2 \rangle$ is

A. $xy^3 + xy^2 + C$

B. $x^3y + x^2y + C$

C. $x^2y^3 + xy^3 + C$

D. $x^2y^3 + x^3 + C$

E. A potential for $\mathbf{F}$ does not exist.

19.(**9 points**) A normal vector to the surface $\mathbf{r}(u, v) = \langle \sin u, \cos u \sin v, \sin v \rangle$ at $(u, v) = (\frac{\pi}{3}, \frac{\pi}{3})$ is

A. $\langle -3, -2, 1 \rangle$

B. $\langle -\frac{1}{8}, \frac{1}{4}, 1 \rangle$

C. $\langle 3, 2, 1 \rangle$

D. $\langle -\frac{3}{8}, \frac{1}{4}, 2 \rangle$

E. $\langle \frac{3}{8}, \frac{1}{4}, 2 \rangle$

20.(**9 points**) Let $\mathbf{F}(x, y, z) = \langle x^2 y, y^2 z, xy^2 \rangle$, then curl $\mathbf{F}(3, 2, 1)$ is

A. $\langle 12, -4, -5 \rangle$

B. $\langle 8, -4, -5 \rangle$

C. $\langle 8, -12, 4 \rangle$

D. $\langle 8, -4, -9 \rangle$

E. $\langle 12, -12, 4 \rangle$

21.(**9 points**) Use Green's theorem to calculate $\int_C y^3 dx - x^3 dy$ where C is the boundary of the disk $\{x^2 + y^2 \leq 9\}$ oriented positively.

A. $\frac{81}{2}\pi$

B. $-\frac{81}{2}\pi$

C. $-\frac{243}{2}\pi$

D. 81π

E. -81π

22.(**10 points**) Let Σ be the part of the sphere $x^2 + y^2 + z^2 = 1$ above the plane $z = \frac{1}{2}$. Find

$$\iint_\Sigma 3z^2 dS.$$

A. $\frac{5\pi}{4}$

B. $\frac{3\pi}{2}$

C. 2π

D. $\frac{7\pi}{4}$

E. $\frac{5\pi}{2}$