MA 26100 | FINAL EXAM | Dec. 19, 2008

Test Number 01

Name: ______________________

Student I.D. #: ______________________

Lecturer: ______________________

Recitation Instructor ______________________

Time of Recitation Class ____________ Sec: ____________

**Instructions:**

1. This exam contains 22 problems worth 9 points each.
2. Please supply all information requested above and on the scantron. Make sure you fill in "Test Number 01" on the scantron.
3. Work only in the space provided, or on the backside of the pages. Mark your answers clearly on the scantron. Also circle your choice for each problem in this booklet.
4. No books, notes, or calculator, please.

**1.** Let $P$ be the plane passing through the origin containing the vectors $\langle 2, 0, 1\rangle$ and $\langle 1, -1, 1\rangle$. For what value of $b$ is the vector $\langle 1, 1, b\rangle$ in the plane $P$ ?

**A.** $b = -1$

**B.** $b = 1$

**C.** $b = -2$

**D.** $b = 2$

**E.** $b = 0$

**2.** Find an equation for the line through the point $(1, 4, -3)$, and parallel to the line $x = 5 + 3t,\ \ y = 1 - t,\ \ z = 1 + 3t$.

**A.** $x = 1 + 5t,\ \ y = 4 + t,\ \ z = -3 + t$

**B.** $x = 3t,\ \ y = -t,\ \ z = 3t$

**C.** $x = 1 + 3t,\ \ y = 4 - t,\ \ z = -3 + 3t$

**D.** $x = 5 + t,\ \ y = 1 + 4t,\ \ z = 1 - 3t$

**E.** $x = 1 - 3t,\ \ y = 4 + 3t,\ \ z = 3 + t$

**3.** A vector parallel to the line of intersection of the planes $x - 2y - z = 6$ and $3x - y + z = 4$ is

**A.** $\langle 3, 4, -5\rangle$

**B.** $\langle 3, 1, -1\rangle$

**C.** $\langle 2, 4, -3\rangle$

**D.** $\langle 3, 4, -1\rangle$

**E.** $\langle 5, -4, -1\rangle$

**4.** Find the point on the upper part of the ellipsoid $x^2 + 2y^2 + z^2 = \dfrac{5}{8}$ at which the normal is parallel to $\langle 1, 1, 1\rangle$.

**A.** $(\sqrt{\frac{5}{8}}, 0, 0)$

**B.** $((0, \sqrt{\frac{5}{16}}, 0)$

**C.** $(\sqrt{\frac{1}{8}}, 0, \sqrt{\frac{1}{2}})$

**D.** $(\frac{1}{2}, \frac{1}{4}, \frac{1}{2})$

**E.** $(0, 0, \sqrt{\frac{5}{8}})$

**5.** An equation of the tangent plane to $z = xy$ at $(1, 2, 2)$ is

**A.** $z = 2x + y + 2$

**B.** $z = \frac{1}{2}(x + y)$

**C.** $z = x - y + 2$

**D.** $z = 2y - x + 1$

**E.** $z = 2x + y - 2$

**6.** Let $f(x, y) = \ln(2x - y)$. Using differentials, the approximate value of $f(1.01, 0.95)$ is

**A.** 0.05

**B.** 0.07

**C.** 0.1

**D.** 0.02

**E.** 0.25

**7.** Let $z = f(x, y)$ be differentiable, and let $x = s + t$, $y = st$.
Then $\dfrac{\partial z}{\partial s}$ at $s = 1, t = 2$

**A.** 1

**B.** $-1$

**C.** 0

**D.** $1/2$

**E.** need more information.

**8.** The maximum rate of change of $f(x, y) = x^2 + xy$ at $(1, 2)$ is in the direction of the vector

**A.** $2\mathbf{i} + \mathbf{j}$

**B.** $\mathbf{i} + 2\mathbf{j}$

**C.** $4\mathbf{i} + \mathbf{j}$

**D.** $\mathbf{i} + \mathbf{j}$

**E.** $5\mathbf{i} + \mathbf{j}$

**9.** Let $M$ be the maximum and $m$ be the minimum of $f(x,y) = xy$ on $x^4 + y^4 = 1$. Then $(M, m) =$

**A.** $(\sqrt{\frac{1}{2}}, -\sqrt{\frac{1}{2}})$

**B.** $(1, 0)$

**C.** $(1, -1)$

**D.** $(\frac{1}{2}, -\frac{1}{2})$

**E.** $(\sqrt{\frac{1}{2}},, -1)$

**10.** Let $\gamma$ be a wire in the shape of the curve $\mathrm{r}(t) = \langle 2t, 1, t^2 \rangle$, $0 \leq t \leq \sqrt{3}$ with mass $\rho(x,y,z) = 3x$. Then the mass of $\gamma$ is

**A.** $2\sqrt{3}$

**B.** $8$

**C.** $4\sqrt{3}$

**D.** $12$

**E.** $28$

**11.** Interchange the limits of integration and evaluate

$$\int_0^4 \int_{\sqrt{y}}^2 \frac{ye^{x^2}}{x^3} dxdy.$$

**A.** $(e^4 - 1)/8$

**B.** $e^4 - 1$

**C.** $(e^4 - 1)/4$

**D.** $e^4/4$

**E.** $(e^4 - 1)/2$

**12.** Let $\Omega$ be the region inside the sphere $x^2 + y^2 + z^2 = 25$ and above the cone $z = \sqrt{x^2 + y^2} + 1$. Then the volume of $\Omega$ is

**A.** $\int_0^{2\pi} \int_0^1 \int_{r+1}^{\sqrt{25-r^2}} r dz dr d\theta$

**B.** $\int_0^{2\pi} \int_0^3 \int_{r+1}^{\sqrt{25-r^2}} r dz dr d\theta$

**C.** $\int_0^{2\pi} \int_0^5 \int_{r+1}^{\sqrt{25-r^2}} r dz dr d\theta$

**D.** $\int_0^{2\pi} \int_0^2 \int_{r+1}^{\sqrt{25-r^2}} r dz dr d\theta$

**E.** $\int_0^{2\pi} \int_0^4 \int_{r+1}^{\sqrt{25-r^2}} r dz dr d\theta$

**13.**

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{-\sqrt{4-x^2-y^2}}^{0} z^2 dzdydx =$$

**A.** $\int_0^{\pi/2}\int_0^{\pi}\int_0^2 \rho^3\cos^2\phi\sin\phi\, d\rho d\theta d\phi$

**B.** $\int_{\pi/2}^{\pi}\int_0^{\pi}\int_0^2 \rho^3\cos^2\phi\sin\phi\, d\rho d\theta d\phi$

**C.** $\int_{\pi/2}^{\pi}\int_0^{\pi}\int_0^2 \rho^4\cos^2\phi\sin\phi\, d\rho d\theta d\phi$

**D.** $\int_0^{\pi/2}\int_0^{\pi}\int_0^2 \rho^4\cos^2\phi\sin\phi\, d\rho d\theta d\phi$

**E.** $\int_0^{\pi/2}\int_0^{2\pi}\int_0^2 \rho^4\cos^2\phi\sin\phi\, d\rho d\theta d\phi$

**14.** Let $\Omega$ be the portion of the disk $\{x^2+y^2\leq 1\}$ lying in the first quadrant, having mass density $\rho(x,y)=x$. If the mass of $\Omega$ is $1/3$, find the $y-$coordinate of the center of mass of $\Omega$.

**A.** 3/7

**B.** 3/5

**C.** 1/2

**D.** 3/8

**E.** 3/4

**15.** Let $D$ be the region bounded by $x = y^2$ and $x = 3y + 4$. Then the area of $D$ is

**A.** $\int_1^{16} \int_{\sqrt{x}}^{(x-4)/3} dydx$

**B.** $\int_{-1}^{4} \int_{3y+4}^{y^2} dxdy$

**C.** $\int_1^{4} \int_{y^2}^{3y+4} dxdy$

**D.** $\int_{-1}^{4} \int_{y^2}^{3y+4} dxdy$

**E.** $\int_1^{16} \int_{(x-4)/3}^{\sqrt{x}} dydx$

**16.** Let $S$ be the surface $z = 6 - 3y - 2x$ lying above the quarter-plane $\{x > 0, y > 0\}$. Then the area of $S$ is

**A.** 18

**B.** $3\sqrt{14}$

**C.** $3\sqrt{13}$

**D.** $6\sqrt{14}$

**E.** 39

**17.** Find the work done by the force field $\mathbf{F}(x, y) = \langle y + x, y - x \rangle$ on a particle that moves in the $xy$−plane along the graph of the function $f(x) = x^2$ from $(0, 0)$ to $(2, 4)$.

A. 7/3

B. 8/3

C. 17/3

D. 22/3

E. 27/3

**18.** Use Green's theorem to evaluate $\int_C x^2\, dy$ where $C$ is the boundary of the rectangle with vertices $\{(0, 0), (2, 0), (2, 3), (0, 3)\}$, oriented conterclockwise.

A. 4

B. 6

C. 8

D. 12

E. 16

**19.** If $S$ is parameterized by $\mathbf{r}(u, v) = \langle u, v, uv^2 \rangle$ then a normal to $S$ at $\langle 1, 2, 4 \rangle$ is

**A.** $\langle 1, 2, 2 \rangle$

**B.** $\langle 1, 2, 4 \rangle$

**C.** $\langle 4, 4, 1 \rangle$

**D.** $\langle 1, 2, -2 \rangle$

**E.** $\langle 4, 4, -1 \rangle$

**20.** Given $\mathbf{F}(x, y, z) = \langle y, z^2 - x, x \rangle$, use Stokes' theorem to determine the flux integral $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$, where $S$ is the graph of $z = 9 - x^2 - y^2$ for $z \geq 0$, oriented by the upward normal.

**A.** $-18\pi$

**B.** $9\pi$

**C.** $18\pi$

**D.** $-9\pi$

**E.** $0$

**21.** Evaluate the flux integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = \langle x, y, z \rangle$ and $S$ is the part of the paraboloid $z = 1 - x^2 - y^2$ in the first octant oriented by the downward pointing normal.

**A.** $\pi/8$

**B.** $-3\pi/8$

**C.** $3\pi$

**D.** $-\pi$

**E.** $0$

**22.** Calculate the flux integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where

$$\mathbf{F}(x, y, z) = 2xy^2\,\mathbf{i} + 2yx^2\,\mathbf{j} - (x^2 + y^2)z\,\mathbf{k}$$

and $S$ is the surface of the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = -1$ and $z = 1$, oriented by the outward normal.

**A.** $\pi$

**B.** $2\pi$

**C.** $3\pi$

**D.** $-\pi$

**E.** $-2\pi$