MA 261 — **TEST NUMBER 1** — **FINAL EXAM** — **December 10, 2007**

Name: ______________________________

Student ID Number: ______________________________

Lecturer: ______________________________

Recitation Instructor: ______________________________

Instructions:

1. This exam contains 22 problems worth 9 points each.
2. Please supply all information requested above and on the mark-sense sheet. Be sure to fill in the "test number" from the top of the page as well.
3. Work only in the space provided, or on the backside of the pages. Mark your answers clearly on the scantron. Also circle your choice for each problem in this booklet.
4. No books, notes, or calculator, please.

Key: beca adbd adec
abcd cdce [redacted] be

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1. The curve $\mathbf{r}_1(t) = \langle t^2 - t,\, t^3 - t,\, t\rangle$ and the line $\mathbf{r}_2(t) = \langle 0,0,1\rangle + t\,\langle 2,1,-1\rangle$ both intersect at the point $(0,0,1)$. The acute angle θ between the curve and the line at $(0,0,1)$ is

A. $\theta = \frac{\pi}{6}$

B. $\theta = \frac{\pi}{3}$

C. $\theta = \frac{\pi}{4}$

D. $\theta = \frac{\pi}{2}$

E. $\theta = \cos^{-1}\left(-\frac{4}{\sqrt{18}}\right)$

2. If the acceleration of an object is $\mathbf{a} = (3t^2 + 2t)\mathbf{i} + 3\,\mathbf{j}$ and its initial velocity is $\mathbf{v}(0) = \mathbf{i} + \mathbf{j}$, what is its speed at $t = 1$?

A. $\sqrt{2}$

B. $\sqrt{13}$

C. $2\sqrt{13}$

D. 10

E. 5

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3. The directional derivative of $f(x, y) = x^2 - y^4 + 1$ at $(1, 1)$ in the direction from $(1, 1)$ to $(2, 0)$ is

A. $\frac{2}{\sqrt{2}}$

B. $\frac{4}{\sqrt{2}}$

C. $\frac{6}{\sqrt{2}}$

D. $\frac{8}{\sqrt{2}}$

E. $\sqrt{20}$

4. Let $w = f(u, v)$ where $\dfrac{\partial f}{\partial u} = 2ve^{2u}$ and $\dfrac{\partial f}{\partial v} = e^{2u}$. If $w(x, y) = f(x^2 - y^2, 3y)$, then at the point $(x_0, y_0) = (1, 1)$, we have $\dfrac{\partial w}{\partial y} =$

A. -9

B. -7

C. 0

D. $-e^2$

E. $-3e^2$

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5. An equation of the tangent plane to the surface $2x^2 + y^2 + ze^z = 3$ at the point $(1, -1, 0)$ is

A. $z = -4x + 2y + 6$

B. $z = -4x - 2y + 6$

C. $z = 4x + 2y - 6$

D. $z = -4x - 2y - 6$

E. $z = 4x - y + 6$

6. If the function u is defined implicitly as a function of x and y by the equation

$$x^3y + xu^2 = 4\sin u + 8,$$

then $\dfrac{\partial u}{\partial x} =$

A. $-\left(\dfrac{x^3y + u^2}{2xu - 4\sin u}\right)$

B. $-\left(\dfrac{2xu}{2xu - 4\cos u}\right)$

C. $\left(\dfrac{x^3y + 2u}{2xu - 4\cos u}\right)$

D. $-\left(\dfrac{3x^2y + u^2}{2xu - 4\cos u}\right)$

E. $-\left(\dfrac{3x^2y + u^2}{2xu}\right)$

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7. Given that the function g has continuous first and second partials and given the table of values below, classify the critical points $(1,1)$ and $(3,0)$ of g:

(x,y)	$g_x(x,y)$	$g_y(x,y)$	$g_{xx}(x,y)$	$g_{yy}(x,y)$	$g_{xy}(x,y)$
$(1,1)$	0	0	-2	-2	-1
$(3,0)$	0	0	0	-6	-3

A. A local minimum at $(1,1)$ and a saddle point at $(3,0)$

B. A local maximum at $(1,1)$ and a saddle point at $(3,0)$

C. A local maximum at $(1,1)$ and a local minimum at $(3,0)$

D. A local minimum at $(1,1)$ and a local maximum at $(3,0)$

E. A saddle point at $(1,1)$ and a local minimum at $(3,0)$

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8. Find the minimum value of

$$f(x, y, z) = 4x^2 + y^2 + z^2$$

on the surface $xyz = 4$.

A. 6

B. 9

C. 10

D. 12

E. 16

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9. Let E be the solid that lies within the cylinder $x^2 + y^2 = 1$, above the plane $z = 0$, and below the paraboloid $z = 4x^2 + 4y^2$. Express

$$\iiint_E x^2 dv$$

as an iterated integral in cylindrical coordinates.

A. $\int_0^{2\pi} \int_0^1 \int_0^{4r^2} r^3 \cos^2 \theta dz dr d\theta$

B. $\int_0^{2\pi} \int_0^4 \int_0^{2r} r^3 \cos^2 \theta dz dr d\theta$

C. $\int_0^{2\pi} \int_0^1 \int_0^{4r^2} r^2 \cos^2 \theta dz dr d\theta$

D. $\int_0^{2\pi} \int_0^2 \int_0^{4r} r^3 \cos^2 \theta dz dr d\theta$

E. $\int_0^{2\pi} \int_0^1 \int_0^4 r^3 \cos^2 \theta dz dr d\theta$

10. Find the values a, b which give the correct change in order of integration

$$\int_5^9 \int_{-\sqrt{9-y}}^{\sqrt{9-y}} f(x,y) dx dy = \int_{-2}^2 \int_a^b f(x,y) dy dx$$

A. $a = 0 \quad b = \sqrt{9 - x^2}$

B. $a = 0 \quad b = 9 - x^2$

C. $a = 9 - x^2 \quad b = 9$

D. $a = 5, \quad b = 9 - x^2$

E. $a = \sqrt{9 - x^2} \quad b = 9$

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11. Find the volume of the solid below the plane $z = x$ and above the triangle with vertices $(0,0,0)$ $(0,3,0)$ and $(3,3,0)$.

A. $1/2$

B. $2/3$

C. $14/3$

D. $3/2$

E. $9/2$

12. Find the surface area of the portion of the paraboloid

$$z = 16 - x^2 - y^2$$

that lies above the disk $x^2 + y^2 \leq 2$.

A. $\pi/2$

B. $7\pi/6$

C. $13\pi/3$

D. $2\pi/3$

E. π

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13. A lamina in the shape of the rectangle $0 \le x \le \pi/2,\ 0 \le y \le 1$, has density $f(x,y) = x\cos(xy)$. Find the mass of the plate.

A. 1

B. π

C. $1/2$

D. $3\pi - 5$

E. $\pi/4$

14. A plane contains the points $P_1(0,1,0)$, $P_2(0,3,3)$, $P_3(1,1,4)$. One vector normal to the plane is:

A. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

B. $8\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$

C. $2\mathbf{j} + 3\mathbf{k}$

D. $\mathbf{i} + 2\mathbf{j} + 7\mathbf{k}$

E. $4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$

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15. Determine a so that the line

$$x = at + 3$$
$$y = 2t - 5$$
$$z = 4t - 1$$

is parallel to the plane $2x + 3y - 5z = 14$.

A. $a = -4$

B. $a = 3$

C. $a = 7$

D. $a = -5$

E. $a = -14$

16. In spherical coordinates, the surface

$$\rho^2(1 - \cos^2\phi) = 16$$

is a:

A. half cone

B. plane

C. sphere

D. cylinder

E. hyperboloid

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17. The traces of the surface $z^2 + 1 = x^2 - y^2$ in the planes $z = k$ are:

A. hyperbolas and lines

B. parabolas only

C. hyperbolas only

D. parabolas and lines

E. hyperbolas and the point $(0, 0)$ only

18. The arclength of the curve given by $\mathbf{r}(t) = \langle \sqrt{2}\, t, e^t, e^{-t} \rangle$ for $0 \le t \le 2$ is:

A. $e^6 - e^{-2}$

B. $e^6 - e^{-1}$

C. $e^4 - e^{-2}$

D. $e^2 - e^{-2}$

E. $2 + e^4 + e^{-4}$

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19. Let C be the curve $y = \frac{1}{3}x^3$, $0 \le x \le 1$. Compute $\int_C 12y\,ds$.

A. $8(2^{3/2} - 1)$

B. $12(2^{3/2} - 1)$

C. $\frac{2}{3}(2^{3/2} - 1)$

D. $\frac{3}{2}(2^{3/2} - 1)$

E. $2(2^{3/2} - 1)$

20. Let C be the segment from $(0, 2)$ to $(2, 5)$. Compute

$$\int_C \frac{1}{2}xy\,dx - x\,dy$$

A. 2

B. $\frac{3}{2}$

C. $\frac{5}{2}$

D. $\frac{4}{3}$

E. 1

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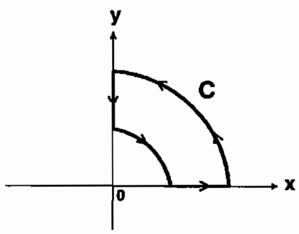
21. Let C be the closed curve pictured that is the boundary of the region in the first quadrant satisfying $1 \leq x^2 + y^2 \leq 4$. Compute

$$\int_C (x - x^2 y)dx + xy^2 dy$$



A. $\frac{7\pi}{6}$

B. $\frac{15\pi}{8}$

C. $\frac{9\pi}{4}$

D. $\frac{11\pi}{2}$

E. $\frac{7\pi}{2}$

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22. Let S be the surface described by $\mathbf{r}(s,t) = (2s-t, s+t, s-t)$ for $0 \le s \le 2,\ 0 \le t \le 1$. Compute

$$\int\int_S (y+z)\,dS$$

A. $2\sqrt{14}$

B. $\sqrt{14}$

C. 2

D. 4

E. $4\sqrt{14}$