MATH 261 - FALL 2002

FINAL EXAM

Name ______________________________ TA ______________________________

FINAL EXAM INSTRUCTIONS

1. There are 20 multiple choice questions each worth 10 points.
2. Blacken the circle on the mark-sense sheet corresponding to your choice of the correct answer.
3. Calculators or books are not permitted.
4. At the end of the examination give both the test booklet and the mark-sense sheet to your TA.

Name ____________________

1. For a smooth curve given by the vector function $\mathbf{r}(t)$, $\mathbf{r}(1) = 3\mathbf{i}+\mathbf{j}$ and $\mathbf{r}(1.25) = 3.5\mathbf{i}+\mathbf{j}$. Then $\mathbf{r}'(1) \approx$

A. $2\mathbf{i}$

B. $0.5\mathbf{i}$

C. $0.25\mathbf{i} + 0.5\mathbf{j}$

D. $-0.5\mathbf{i} + 0.5\mathbf{j}$

E. $-0.25\mathbf{i} + 0.5\mathbf{j}$

2. The cosine of the angle between the two planes $x - y + 2z = 7$ and $2x + y - z = 5$ is

A. $\frac{1}{6}$

B. $-\frac{1}{\sqrt{6}}$

C. $\frac{1}{3}$

D. $-\frac{1}{6}$

E. $\frac{2}{\sqrt{6}}$

Name ______________________________

3. The line tangent to the curve

$$\mathbf{r}(t) = \langle \sin(4\pi t), \cos(2\pi t), t^2 \rangle$$

at $t = \dfrac{1}{4}$ is the intersection of which two planes? (Hint: Find symmetric equations of the tangent line.)

A. $y = 2x,\ y = 4\pi z - \dfrac{\pi}{8}$

B. $y = 2x,\ y = 4\pi z - \dfrac{\pi}{2}$

C. $y = \dfrac{1}{2}x,\ y = -4\pi z + \dfrac{\pi}{4}$

D. $y = -\dfrac{1}{2}x,\ y = 2\pi z - \dfrac{\pi}{4}$

E. $y = \dfrac{1}{2}x,\ y = 2\pi z + \dfrac{\pi}{4}$

4. The length of the curve $\mathbf{r}(t) = \langle 2t, t^2, \ln t \rangle$, $1 \le t \le 2$ is

A. $4 + \ln 2$

B. $3 + \ln 2$

C. $7 + \ln 2$

D. $3 + e$

E. $7 + e$

Name ______________________________

5. Evaluate $\lim\limits_{(x,y)\to(0,0)} \dfrac{5xy^2}{x^2+y^2}$.

A. Does not exist

B. 5

C. $\dfrac{5}{2}$

D. 1

E. 0

6. Find an equation for the tangent plane at $(1, 1, -1)$ of the surface

$x^2 + 2xy + y^2 + z^3 = 3$.

A. $4x + 4y + 3z = 5$

B. $3x - 2y + 4z = -3$

C. $4x + y - z = 6$

D. $3x + 4y + 2z = 5$

E. $x + 6y - 2z = 9$

Name ____________________

7. Find an equation for the line through $(e, 1)$ which is orthogonal to the level curve of $z = \ln(xy^3)$ that contains $(e, 1)$.

A. $2x - 4y = 2e - 4$

B. $y = 3e(x - e) + 1$

C. $x + 3ey = 4e$

D. $y = 3(x - e) + 1$

E. $x - 3y = e - 3$

8. Find the maximum rate of change of

$$f(x, y) = x^2e^y + 3xy$$

at the point $(2, 0)$.

A. $\sqrt{13}$

B. $2\sqrt{26}$

C. 12

D. $2\sqrt{29}$

E. $3\sqrt{7}$

Name ______________________________

9. Use the method of Lagrange multipliers to find the maximum value of $f(x,y) = x^2y$ subject to the constraint $2x^2 + 3y^2 = 1$.

A. $\dfrac{1}{5\sqrt{5}}$

B. $\dfrac{1}{9}$

C. $\dfrac{2}{3\sqrt{3}}$

D. 2

E. 1

10. The region R in the xy-plane is the triangle with vertices $(0,0)$, $(2,0)$, and $(2,1)$. Evaluate

$$\iint_R y\,dA\ .$$

A. 2

B. $\dfrac{8}{3}$

C. $\dfrac{2}{3}$

D. 1

E. $\dfrac{1}{3}$

Name ______________________________

11. Find a and b for the correct interchange of the order of integration:

$$\int_0^4 \int_x^{2\sqrt{x}} f(x,y)dydx = \int_0^4 \int_a^b f(x,y)dxdy.$$

A. $a = y,\ b = 4y^2$

B. $a = y,\ b = \dfrac{\sqrt{y}}{2}$

C. $a = \dfrac{y^2}{4},\ b = y$

D. $a = \dfrac{\sqrt{y}}{2},\ b = y$

E. cannot be done without knowing $f(x,y)$

12. The double integral $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} y^2(x^2+y^2)^3 dydx$ when converted to polar coordinates becomes

A. $\displaystyle\int_0^{\pi} \int_0^1 r^9 \sin^2\theta drd\theta$

B. $\displaystyle\int_0^{\frac{\pi}{2}} \int_0^1 r^9 \sin^2\theta drd\theta$

C. $\displaystyle\int_0^{\pi} \int_0^1 r^8 \sin\theta drd\theta$

D. $\displaystyle\int_0^{\frac{\pi}{2}} \int_0^1 r^8 \sin\theta drd\theta$

E. $\displaystyle\int_0^{\frac{\pi}{2}} \int_0^1 r^8 \sin^2\theta drd\theta$

Name ____________________

13. The solid region Q is bounded by the surfaces $x^2 + y^2 = 1$, $y + z = 2$, and $z = 0$. Express the volume of the solid as an iterated triple integral in cylindrical coordinates.

A. $\int_0^{2\pi} \int_0^1 \int_0^{2-r\sin\theta} r\, dzdrd\theta$

B. $\int_0^{2\pi} \int_0^{\sin\theta} \int_0^2 r\, dzdrd\theta$

C. $\int_0^{\pi} \int_0^{\sin\theta} \int_0^2 r\, dzdrd\theta$

D. $\int_0^{2\pi} \int_0^{\sin\theta} \int_0^{2-r\sin\theta} r\, dzdrd\theta$

E. $\int_0^{\pi} \int_0^1 \int_0^{2-r\sin\theta} r\, dzdrd\theta$

14. The integral in spherical coordinates

$$\int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_0^{\frac{1}{\sin\varphi}} \rho^2 \sin\varphi d\rho d\varphi d\theta$$

when converted to rectangular coordinates becomes

A. $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^1 dzdydx$

B. $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \int_1^{\sqrt{x^2+y^2}} dzdydx$

C. $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{x^2+y^2}} dzdydx$

D. $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{x^2+y^2} dzdydx$

E. $\int_0^1 \int_0^{1-x^2} \int_0^{\sqrt{x^2+y^2}} dzdydx$

Name ________________________________

15. The area of the surface of the cone $z^2 = x^2 + y^2$ between the planes $z = 1$, $z = 4$ is

A. 30π

B. $(\sqrt{2})(15\pi)$

C. 15π

D. 16π

E. 4π

16. Evaluate $\int_C y\,dx + x\,dy$ where C consists of the line segments from $(0,0)$ to $(1,0)$ and from $(1,0)$ to $(1,2)$.

A. 0

B. $\frac{2}{3}$

C. $\frac{5}{6}$

D. 2

E. 3

17. Evaluate $\iint_S x\,dS$ where S is the portion of the plane $x+y+z=1$ in the first octant.

A. $\sqrt{3}$

B. $\frac{\sqrt{3}}{2}$

C. $\frac{\sqrt{3}}{6}$

D. $\frac{\sqrt{3}}{3}$

E. $\frac{\sqrt{3}}{4}$

18. If $(2,1,a)$ is a point on the tangent plane to the parametric surface

$$\mathbf{r}(u,v) = (u+v^2)\mathbf{i} + (u-v)\mathbf{j} + (u-v^2)\mathbf{k}$$

at $\mathbf{r}(1,1) = \langle 2,0,0\rangle$, then $a =$

A. $\frac{4}{3}$

B. 1

C. 2

D. 3

E. -1

19. Let C be the intersection of the surfaces $x^2 + y^2 = 1$ and $x + y + z = 10$. If C is oriented counterclockwise, use Stoke's Theorem to evaluate

$$\int_C y\,dx + z\,dy + x\,dz.$$

A. π

B. 2π

C. 4π

D. 0

E. -3π

20. Use the divergence theorem to evaluate the flux integraal

$$\iint_S \mathbf{F} \cdot \mathbf{n}\,dS$$

where $\mathbf{F} = yz^2\mathbf{i} + x^2z^3\mathbf{j} + 2z\mathbf{k}$, S is the sphere $x^2 + y^2 + z^2 = \frac{1}{4}$, and $\mathbf{n}$ is the outward unit normal.

A. $\frac{\pi}{3}$

B. $\frac{4\pi}{3}$

C. 4π

D. 8π

E. $\frac{8\pi}{3}$