MATH 261 – FALL 2000 – FINAL EXAM

STUDENT NAME ——————————————————-

STUDENT ID ——————————————————-

RECITATION HOUR ——————————————————-

RECITATION INSTRUCTOR ——————————————

INSTRUCTOR ——————————————————-

INSTRUCTIONS

1. This test booklet has 14 pages including this one. There are 25 questions, each worth 8 points.

2. Fill in your name, your ID number, your recitation hour, the name of your recitation instructor and the name of your instructor above.

3. Use a number 2 pencil on the mark-sense sheet (answer sheet) to do the following:

3.1. On the top left side, write your name (last name, first name) and fill in the little circles.

3.2 On the bottom left side, under SECTION, write in your division and section number and fill in the little circles.

3.3. On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your student ID number, and fill in the little circles.

3.4 Blacken your choice of the correct answer in the spaces provided for questions 1-25.

4. After you have finished the exam, turn in BOTH the answer sheet and the question sheets to your instructor.

5. No books or notes or calculators may be used.

1) Which of the following is an equation for the plane that contains the point $(3, 2, 1)$ and is perpendicular to the line with vector equation

$$\vec{r}(t) = (1 - 2t)\,\vec{i} + 3t\,\vec{j} + (4 + t)\,\vec{k}?$$

A) $2x - 3y - z = -1$

B) $2x + 3y + z = 14$

C) $x + 2y - z = 8$

D) $2x - y + z = 0$

E) $x + y + z = 4$

2) Which of the following are parametric equations for the line that passes throug $(1, 1, 2)$ and is parallel to the line $\frac{x-3}{2} = \frac{y-1}{3} = z - 4$.

A) $x(t) = 1 + 2t, \quad y(t) = 1 + 2t, \quad z(t) = 2 + t$

B) $x(t) = 1 + 2t, \quad y(t) = 1 + 3t, \quad z(t) = 2 + t$

C) $x(t) = 1 + t, \quad y(t) = 1 + 2t, \quad z(t) = 2 + 3t$

D) $x(t) = 1 + 4t, \quad y(t) = 1 + 3t, \quad z(t) = 2 + 3t$

E) $x(t) = 1 + 2t, \quad y(t) = 1 + 3t, \quad z(t) = 2 + 2t$

3) A particle is moving in space with constant acceleration $\vec{a}(t) = 2\,\vec{i}$. Its initial position was $\vec{r}(0) = \vec{j}$ and its initial velocity was $\vec{v}(0) = \vec{j}$. When does the particle cross the plane $y = 3$?

A) $t = 1$

B) $t = \frac{1}{2}$

C) $t = 2$

D) $t = 3$

E) never

4) The level surfaces of the functions

$$F(x, y, z) = x^2 + y^2 + z^2 - 2z, \quad G(x, y, z) = z - x^2 - y^2,$$
$$\text{and } H(x, y, z) = x^2 + y^2$$

are respectively

A) Spheres, paraboloids and circles

B) Cones, paraboloids and hyperboloids

C) Spheres, paraboloids and cylinders

D) Paraboloids, cones and hyperboloids

E) Paraboloids, spheres and cones.

5) The curl of the vector field $\vec{F}(x,y,z) = x\,\vec{i} + y\,\vec{j} + xyz\,\vec{k}$ is

A) $x\,\vec{i} + y\,\vec{j} + xyz\,\vec{k}$

B) $xz\,\vec{i} - yz\,\vec{j}$

C) $-xz\,\vec{i} + yz\,\vec{j}$

D) $x^2\,\vec{i} + y^2\,\vec{j} + z^2\,\vec{k}$

E) $xyz\,\vec{i} + 2xy\,\vec{j} + 3z\,\vec{k}$

6) Let $f(x,y,z) = x^3 + \sqrt{6}y^2 + z^4$. The directional derivative of f at the point $(1,1,1)$ in the direction in which f increases most rapidly is

A) 2

B) 3

C) 5

D) 7

E) 4

7) Let $f(x, y) = \sin(xy)$. Find $\frac{\partial^2 f}{\partial x \partial y}$.

A) $\cos(xy)$

B) $\cos(xy) + xy\sin(xy)$

C) $-xy\sin(xy)$

D) $\sin(xy) - \cos(xy)$

E) $\cos(xy) - xy\sin(xy)$

8) Find an equation of the tangent plane to the surface $z = \sqrt{x^2 + y^2}$ at the point $(3, -4, 5)$.

A) $2x + y - 5z = -23$

B) $4x - y + z = 0$

C) $3x - 4y + 5z = 0$

D) $3x + y + z = 10$

E) $3x - 4y - 5z = 0$

9) Let $u(x,y) = x^2 - xy$, $x = st$, $y = s^2 - t^2$. Find $\frac{\partial u}{\partial s}$ at $(s,t) = (1,3)$.

A) 36

B) 42

C) 24

D) 48

E) 54

10) If $xyz + z^2 = 15$ determines z implicitly as a function of x and y, $\frac{\partial z}{\partial x}$ at the point $(2,1,3)$ is

A) 0

B) -1

C) $-\frac{1}{4}$

D) $-\frac{3}{8}$

E) $-\frac{3}{4}$

11) The function $f(x,y) = x^3 - x - y^2 + 2y$ has

A) Two relative minima

B) A relative maximum and a relative minimum

C) Two saddle points

D) A relative maximum and a saddle point

E) A relative minimum and a saddle point

12) Find the point (x,y), which satisfies $y - x^2 = 0$, at which $f(x,y) = (x-16)^2 + (y - \frac{1}{2})^2$ is minimum.

A) $(1,1)$

B) $(2,4)$

C) $(3,9)$

D) $(\frac{1}{2}, \frac{1}{4})$

E) $(6,36)$

13) Evaluate $\int\int_D \sin\left(\frac{\pi}{2}x^2\right)\ dA$, where D is the region of the plane bounded by $x = y$, $y = 0$ and $x = 1$.

A) π

B) $\frac{1}{\pi}$

C) $\frac{2}{\pi}$

D) 2π

E) 3π

14) The area of the region inside the circle $r = 2\cos\theta$ and above the line $y = \sqrt{3}x$ is given by

A) $\int_0^{\frac{\pi}{2}} \int_0^{2\cos\theta} r\ dr\ d\theta$

B) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \int_0^{2\cos\theta} r\ dr\ d\theta$

C) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \int_0^{2\cos\theta} r\ dr\ d\theta$

D) $\int_0^{\frac{\pi}{3}} \int_0^{2\cos\theta} r\ dr\ d\theta$

E) $\int_0^{\frac{\pi}{6}} \int_0^{2\cos\theta} r\ dr\ d\theta$

15) Let D be the solid region bounded by the planes $x + z = 1$, $x = y$, $y = 0$ and $z = 0$. Then the volume of D is

A) $\frac{1}{2}$

B) $\frac{1}{3}$

C) $\frac{1}{6}$

D) $\frac{1}{8}$

E) 1.

16) Evaluate $\int\int\int_D 5xy \; dV$ where D is the part in the first octant of the solid region bounded by the surfaces $z = \sqrt{x^2 + y^2}$, $z = 0$ and $x^2 + y^2 = 1$.

A) $\frac{5}{2}$

B) $\frac{5}{4}$

C) $\frac{3}{2}$

D) $\frac{1}{2}$

E) 5

17) The surface area of the portion of the hemisphere $z = \sqrt{2 - x^2 - y^2}$ inside the paraboloid $z = x^2 + y^2$ is

A) $\int_0^{2\pi} \int_0^1 \sqrt{1 + r^2}\ r\ dr\ d\theta$

B) $\int_0^{2\pi} \int_0^1 \sqrt{\frac{2}{2-r^2}}\ r\ dr\ d\theta$

C) $\int_0^{2\pi} \int_0^1 \sqrt{\frac{2}{2+r^2}}\ r\ dr\ d\theta$

D) $\int_0^{2\pi} \int_0^1 \frac{2}{\sqrt{2-r^2}}\ r\ dr\ d\theta$

E) $\int_0^{2\pi} \int_0^1 (2 + r^2)\ r\ dr\ d\theta$

18) Compute the line integral $\int_C y \sin x\ dx + y^2\ dy$ along the curve composed of the line segment from $(0, 0)$ to $(2, 0)$ and the line segment from $(2, 0)$ to $(2, 1)$.

A) $\frac{1}{3}$

B) 1

C) $2 \sin 2$

D) $2 \sin 2 + \sin 1 + 1$.

E) $\sin 2 + \sin 1$

19) Evaluate $\int_C xy\ dx + xy\ dy$, where C be the circle $x^2 + y^2 = 4$ oriented counterclockwise.

A) 2π

B) 3π

C) 0

D) $\frac{3\pi}{2}$

E) 2

20) Let $f(x, y, z) = 8x^3y^4 + 9yze^x + y\sin(z^3 + x^4)$. Let $\vec{F}(x, y, z) = \nabla f(x, y, z)$ and let C be parametrized by $\vec{r}(t) = \cos t\ \vec{i} + \sin t\ \vec{j} + t\ \vec{k}$, $0 \leq t \leq \pi$. Then $\int_C \vec{F} \cdot d\vec{r}$ is equal to

A) 3π

B) 2π

C) $-\pi$

D) 1

E) 0.

21) A sheet of metal Σ is formed by the portion of the paraboloid $z = 10 - x^2 - y^2$ inside the cylinder $x^2 + y^2 = 9$. The mass density of the metal at a point (x, y, z) is $\delta(x, y, z) = \frac{1}{\sqrt{41-4z}}$. The mass of the sheet is:

A) π

B) 3π

C) 16π

D) 5π

E) 9π

22) Which of the following gives a unit vector $\vec{n}(x, y, z)$ that is normal to the surface $z = \sqrt{x^2 + y^2}$, $4 \leq x^2 + y^2 \leq 9$, and points downward?

A) $\vec{n}(x, y, z) = x\,\vec{i} + y\,\vec{j} - z\,\vec{k}$

B) $\vec{n}(x, y, z) = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$

C) $\vec{n}(x, y, z) = \frac{1}{\sqrt{2(x^2+y^2)}} \left(x\,\vec{i} + y\,\vec{j} + \sqrt{x^2 + y^2}\,\vec{k} \right)$

D) $\vec{n}(x, y, z) = \frac{1}{\sqrt{2(x^2+y^2)}} \left(x\,\vec{i} + y\,\vec{j} - \sqrt{x^2 + y^2}\,\vec{k} \right)$

E) $\vec{n}(x, y, z) = \frac{1}{\sqrt{2(x^2+y^2)}} \left(-x\,\vec{i} - y\,\vec{j} + \sqrt{x^2 + y^2}\,\vec{k} \right)$

23) Let Σ be the part of the paraboloid $z = x^2 + y^2$ inside the cylinder $x^2 + y^2 = 4$, and let the unit normal $\vec{n}$ on Σ be directed upwards. If $\vec{F}(x, y, z) = x\,\vec{i} - \vec{j} + 2x^2\,\vec{k}$, then $\int\int_{\Sigma} \vec{F} \cdot \vec{n}\, dS$ is equal to.

A) 0

B) $\frac{2\pi}{3}$

C) π

D) $\frac{32\pi}{3}$

E) $\frac{3\pi}{4}$

24) If $\vec{F}(x, y, z) = y^2 z^3\,\vec{i} - xz\,\vec{j} + z\,\vec{k}$, Σ is the sphere $x^2 + y^2 + z^2 = 4$, and $\vec{n}$ is the outward unit normal of Σ, then $\int\int_{\Sigma} \vec{F} \cdot \vec{n}\, dS$ equals

A) $\frac{32\pi}{3}$

B) 16π

C) 8π

D) $\frac{20\pi}{3}$

E) $\frac{16\pi}{3}$

25) Let $\vec{F}(x,y,z) = \cos(y^2z^3)\ \vec{i} - xz^8y^3\ \vec{j} + z^{16}\ \vec{k}$, and let Σ be the ellipsoid $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} = 1$. Let $\vec{n}$ be the outward unit normal of Σ, then $\int\int_{\Sigma}(\text{curl}\ \vec{F}) \cdot \vec{n}\ dS$ equals

A) 1

B) 5

C) 3

D) 2

E) 0