NAME ______________________________

STUDENT ID # ______________________________

INSTRUCTOR ______________________________

INSTRUCTIONS

1. There are 12 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. I.D.# is your 9 digit ID (probably your social security number). Also write your name at the top of pages 2–12.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet.
4. No books, notes or calculators may be used on this exam.
5. Each problem is worth 8 points. The maximum possible score is 200 points.
6. Using a #2 pencil, fill in each of the following items on your answer sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION, write in your division and section number and fill in the little circles. (For example, for division 9 section 1, write 0901. For example, for division 38 section 2, write 3802).
   (c) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your student ID number, and fill in the little circles.
   (d) Using a #2 pencil, put your answers to questions 1–25 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
   (e) Sign your answer sheet.
7. After you have finished the exam, hand in your answer sheet and your test booklet to your instructor.

1. Find a unit vector that is perpendicular to $\vec{a} = \vec{i} - 2\vec{j}$ and $\vec{b} = \vec{i} + \vec{k}$.

A. $\frac{2}{3}\,\vec{i} + \frac{1}{3}\,\vec{j} - \frac{2}{3}\,\vec{k}$

B. $2\vec{i} - \vec{j} - 2\vec{k}$

C. $\frac{2}{5}\vec{i} + \frac{1}{5}\vec{j} - \frac{2}{5}\vec{k}$

D. $2\vec{i} + \vec{j} - 2\vec{k}$

E. $\frac{2}{3}\,\vec{i} + \frac{1}{3}\,\vec{k}$

2. Find parametric equations for the line that contains $(3, -2, 1)$ and that is parallel to the line with equations

$$\frac{x-2}{4} = \frac{y+5}{2} = z - 3$$

A. $x = \frac{t}{4} + 3,\ y = \frac{t}{2} - 2,\ z = t + 1$

B. $x = 3t - 2,\ y = -2t + 5,\ z = t - 3$

C. $x = 4t - 3,\ y = 2t + 2,\ z = t - 1$

D. $x = \frac{t}{4} - 3,\ y = \frac{t}{2} + 2,\ z = t - 1$

E. $x = 4t + 3,\ y = 2t - 2,\ z = t + 1$

3. Find an equation of the plane that contains the point $(2, 0, 1)$ and that is perpendicular to the line through $(1, 2, 1)$ and $(2, 5, 3)$.

A. $x + 2y + z = 3$

B. $2x + 5y + 3z = 7$

C. $x + 3y + 2z = 4$

D. $x + 2y + z = -3$

E. $x + 3y + 2z = -4$

4. If the acceleration of a particle at time t is $\vec{a}(t) = (t+1)\vec{i} + (2t-1)\vec{j} + 3\vec{k}$ and the velocity $\vec{v}(t)$ at time $t = 0$ is $\vec{v}(0) = \vec{i} + 2\vec{k}$, then $\vec{v}(2)$ equals

A. $4\vec{i} + 2\vec{j} + 6\vec{k}$

B. $5\vec{i} + 2\vec{j} + 8\vec{k}$

C. $\frac{3}{2}\,\vec{i} + 3\vec{k}$

D. $\frac{5}{2}\,\vec{i} + 5\vec{k}$

E. $3\vec{i} + 2\vec{j} + 4\vec{k}$

5. The line tangent to the curve $r(\vec{t}) = t\vec{i} + e^t\vec{j} + (\sin t)\vec{k}$ at $t = 0$ is:

A. $x = t,\ y = t,\ z = t$

B. $x = t,\ y = 1 + t,\ z = t$

C. $x = t,\ y = e^t,\ z = \cos t$

D. $x = 1,\ y = e^t,\ z = \cos t$

E. $x = t,\ y = 1 + e^t,\ z = t$

6. Find the length of the curve $r(\vec{t}) = \left(t - \frac{t^3}{3}\right)\vec{i} + t^2\vec{j} + \left(t + \frac{t^3}{3}\right)\vec{k}$ for $0 \le t \le 1$.

A. $1/3$

B. $\sqrt{2}/3$

C. $4/3$

D. $4\sqrt{2}/3$

E. $8/3$

7. If $f(x,y) = 2xye^{(5x^2+y^2)}$, then $\dfrac{\partial f}{\partial x}(1,2) =$

A. $4e^9$

B. $6e^9$

C. $8e^9$

D. $40e^9$

E. $44e^9$

8. The equation of the tangent plane to the graph of the function $f(x,y) = x - \dfrac{y^2}{2}$ at $(1,2,-1)$ is

A. $2x + y + 4z = 0$

B. $x + 4y = 9$

C. $x - 2y - z = 2$

D. $-x + 2y + z = 2$

E. $x - y - 2z = 1$

9. If $z = xe^{y^2}$, $x = u + v$, $y = u^2 - v$, find $\dfrac{\partial z}{\partial u}$ where $u = 2$ and $v = 3$.

A. $2e^2$

B. $45e$

C. $20e^2$

D. $41e$

E. $17e$

10. The function $f(x, y) = 3xy - x^3 + y^3 + 1$ has

A. 1 rel. max. and 1 rel. min.

B. 2 rel. min.

C. 2 saddle pt.

D. 1 saddle pt. and 1 rel. min.

E. 1 saddle pt. and 1 rel. max.

11. Find the minimum of the function $f(x, y) = x^2 + y^2$ subject to the constraint $x^2 - 4x - y^2 = -12$.

A. 10

B. 12

C. 6

D. 8

E. 14

12. Reversing the order of integration $\int_1^2 \int_0^{\ln x} x^2 y\,dy\,dx =$

A. $\int_0^{\ln y} \int_1^2 xy^2\,dx\,dy$

B. $\int_0^{\ln 2} \int_{e^y}^2 x^2 y\,dx\,dy$

C. $\int_1^{\ln 2} \int_2^{e^y} x^2 y\,dx\,dy$

D. $\int_1^{\ln 2} \int_0^{e^y} x^2 y\,dx\,dy$

E. $\int_0^{\ln 2} \int_1^{e^y} x^2 y\,dx\,dy$

13. Find the surface area of the portion of the paraboloid $z = 9 - x^2 - y^2$ above the xy–plane.

A. $\frac{\pi}{6}\left[(35)^{3/2} - 1\right]$

B. $\frac{\pi}{5}\left[(30)^{1/2} - 1\right]$

C. $\frac{\pi}{7}\left[(36)^{3/2} - 1\right]$

D. $\frac{\pi}{6}\left[(37)^{3/2} - 1\right]$

E. $\frac{\pi}{4}\left[(34)^{3/2} - 1\right]$

14. The volume of the solid bounded above by the paraboloid $z = 2 - x^2 - y^2$ and below by the paraboloid $z = x^2 + y^2$ is

A. π

B. $\dfrac{3\pi}{4}$

C. $\dfrac{\pi}{2}$

D. $\dfrac{4\pi}{5}$

E. 2π

15. In spherical coordinates, the equation of the upper nappe of the cone $z^2 = 3(x^2 + y^2)$ is

A. $\varphi = \dfrac{\pi}{4}$

B. $\varphi = \dfrac{\pi}{3}$

C. $\varphi = \dfrac{\pi}{2}$

D. $\varphi = \dfrac{\pi}{6}$

E. $\varphi = 0$

16. An object occupies the portion in the first octant of the solid region bounded above by the sphere $x^2 + y^2 + z^2 = 4$ and below by the cone $z = \sqrt{x^2 + y^2}$. The density at each point of the object is equal to the square of the distance of the point from the origin. Use spherical coordinates to compute the total mass of the object.

A. $\dfrac{64\pi}{7}$

B. $\dfrac{32\pi}{3}\left(1 - \dfrac{\sqrt{3}}{2}\right)$

C. $16\pi\left(1 - \dfrac{\sqrt{2}}{2}\right)$

D. $\dfrac{32\pi}{5}\left(1 - \dfrac{\sqrt{3}}{2}\right)$

E. $\dfrac{16\pi}{5}\left(1 - \dfrac{\sqrt{2}}{2}\right)$

17. An object occupies the region defined by $x \geq 0,\ y \geq 0$, and $x^2 + y^2 + z^2 \leq 4$. If the mass density is given by $\delta(x, y, z) = (x^2 + y^2)^2$, which of the integrals is equal to the total mass of the object.

A. $\displaystyle\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^{2} \rho^6 \sin^3 \phi d\rho d\phi d\theta$

B. $\displaystyle\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^{2} \rho^6 \sin^5 \phi d\rho d\phi d\theta$

C. $\displaystyle\int_0^{\frac{\pi}{2}}\int_0^{\pi}\int_0^{2} \rho^6 \sin^3 \phi d\rho d\phi d\theta$

D. $\displaystyle\int_0^{\frac{\pi}{2}}\int_0^{\pi}\int_0^{2} \rho^4 \sin^5 \phi d\rho d\phi d\theta$

E. $\displaystyle\int_0^{\frac{\pi}{2}}\int_0^{\pi}\int_0^{2} \rho^6 \sin^5 \phi d\rho d\phi d\theta$

18. A potential function for the vector field $\vec{F}(x,y) = 2e^y\vec{\imath} + (2xe^y + y)\vec{\jmath}$ is

A. $2xe^y + \dfrac{y^2}{2} + C$

B. $2xe^y + C$

C. $x^2e^y + xy + C$

D. $2xe^y + yx^2 + C$

E. There is no potential function for this vector field

19. If $\vec{F}(x,y,z) = (y+z)\vec{\imath} + 2(z+x)\vec{\jmath} + (x+y)\vec{k}$, then curl $\vec{F}(x,y,z) =$

A. $\vec{0}$

B. $-\vec{\imath}$

C. $\vec{\imath} + \vec{k}$

D. $\vec{\imath} - \vec{k}$

E. $-\vec{\imath} + \vec{k}$

20. Evaluate $\int\limits_C \vec{F} \cdot d\vec{r}$, where $\vec{F}(x,y) = y^2\vec{i} + x\vec{j}$, and C is the line segment from $(0,0)$ to $(1,2)$.

A. $\frac{5}{2}$

B. $\frac{7}{3}$

C. 2

D. $\frac{3}{2}$

E. $\frac{4}{3}$

21. If C is the complete boundary of the triangle with vertices $(0,0)$, $(1,0)$, $(1,1)$ and C is oriented counterclockwise, then

$$\int_C (y\cos x + 2y)dx + \sin x dy =$$

A. 1

B. 2

C. 4

D. -2

E. -1

22. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where

$$C: \quad x = \cos t,\ y = \sin t,\ z = t \qquad 0 \le t \le \pi$$

and $\vec{F} = \text{grad } f$ with $f(x, y, z) = y^3 + 2xz$.

A. 2π

B. 4

C. 3π

D. 6π

E. -2π

23. If Σ is the portion of the cone $z^2 = x^2 + y^2$ with $0 \le z \le 2$, then $\iint_\Sigma z^2 dS =$

A. 18π

B. $6\sqrt{2}\ \pi$

C. $8\sqrt{2}\ \pi$

D. 6π

E. 4π

24. Suppose Σ is the part of the plane $z = 3$ inside the cylinder $x^2 + y^2 = 1$ and Σ is oriented by the unit normal $\vec{n}$ directed upward. If $F(x, y, z) = x\vec{i} + y\vec{j} + z\vec{k}$ then $\iint_\Sigma \vec{F} \cdot \vec{n} dS =$

A. 3π

B. 4π

C. 6π

D. -3π

E. 8π

25. Let D be the region in the first octant bounded by the planes $x = 1$, $y = 1$ and $z = 2$. If Σ is the complete boundary of D, evaluate $\iint_\Sigma \vec{F} \cdot \vec{n} dS$ where $\vec{F}(x, y, z) = x^2\vec{i} - 2xy\vec{j} + z\vec{k}$ and $\vec{n}$ is the unit normal to Σ directed outward.

A. -2

B. 0

C. 4

D. 2

E. 6