MA 261 EXAM 2 Spring 2003

Name ______________________________

Student ID ______________________________

Recitation Instructor ______________________________

Recitation Time ______________________________

Directions

1. Write your name, student ID number, recitation instructor's name and recitation time in the spaces provided above, and also fill in the information on the answer sheet.
2. Circle the letter of your answer for each question in the test papers and also on the answer sheet.
3. The exam has 12 problems. Problems 1–4 are worth 9 points each. All others are worth 8 points each.
4. No books, notes or calculators may be used in this exam.

1. Given $x^2 + y^2 + \sin(xy^2) = 1$. Find $\dfrac{dy}{dx}$ at $(0, 1)$.

A. 0

B. $\dfrac{1}{2}$

C. 2

D. $-\dfrac{1}{2}$

E. -2

2. Find the directional derivative of $f(x, y) = \tan(x + 2y)$ at the point $\left(0, \dfrac{\pi}{6}\right)$ in direction of $(-3, 4)$.

A. 20

B. $\dfrac{4}{5}$

C. 4

D. 16

E. -20

3. Critical points of $f(x, y) = x^3 + y^3 - 6xy$ are

A. a saddle point and a local maximum

B. a saddle point and a local minimum

C. a local maximum and a local minimum

D. two saddle points and a local maximum

E. two saddle points and a local minimum

4. Use Lagrange multipliers to find the point (x, y, z) at which $x^2 + y^2 + z^2$ is minimal subject to $x + 2y + 3z = 1$.

A. $\left(\frac{1}{7}, \frac{1}{14}, \frac{2}{7}\right)$

B. $\left(\frac{1}{14}, \frac{1}{7}, \frac{3}{14}\right)$

C. $(1, 0, 0)$

D. $\left(\frac{1}{2}, \frac{1}{4}, 0\right)$

E. $\left(\frac{3}{14}, \frac{1}{7}, \frac{1}{14}\right)$

5. What is the double integral for the volume of the solid in the first octant bounded by the surfaces $z^2 = x^2 + 2y^2$, $x + y = 1$.

A. $\int_0^1 \int_0^{1-x} (x^2+2y^2)dydx$

B. $\int_0^1 \int_0^{1-x} (x^2+2y^2)^2 dydx$

C. $\int_0^1 \int_0^{1-y} \sqrt{x^2 + 2y^2}dydx$

D. $\int_0^1 \int_0^{1+x} \sqrt{x^2 + 2y^2}dydx$

E. $\int_0^1 \int_0^{1-x} \sqrt{x^2 + 2y^2}dydx$

6. Evaluate $\iint_R \frac{yx^2}{1+y^2}\, dA$, $R = [-3, 3] \times [0, 1]$.

A. $\frac{9}{2}\ln 2$

B. 0

C. 18

D. $9 \ln 2$

E. $18 \ln 2$

7. Evaluate $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\,dxdy$.

A. $\frac{1}{3}\left(2^{\frac{3}{2}}-1\right)$

B. $\frac{2}{3}\left(2^{\frac{3}{2}}-1\right)$

C. cannot be evaluated

D. $\frac{2}{3}$

E. $\frac{2}{9}\left(2^{\frac{3}{2}}-1\right)$

8. Evaluate $\iint_R xy\,dxdy$ where R is the region in the first quadrant that lies between the circles $x^2+y^2=1$ and $x^2+y^2=9$.

A. 10

B. 10π

C. 20

D. $\frac{13}{3}$

E. $\frac{13}{3}\pi$

9. A lamina occupies the region in the first quadrant bounded by $y = x^2$ and $y = 1$. If density $\rho(x, y) = xy$, the x coordinate of center of gravity equals

A. $\frac{1}{6}$

B. $\frac{1}{2}$

C. $\frac{3}{4}$

D. $\frac{4}{7}$

E. $\frac{2}{21}$

10. Find the area of the part of the surface $z = y^2 - 2x$ that lies above the triangle with vertices $(0, 0)$, $(1, 1)$, and $\left(\frac{1}{2}, 1\right)$.

A. $\frac{1}{24}(27 - 5\sqrt{5})$

B. $\frac{1}{48}(27 - 5\sqrt{5})$

C. $\frac{1}{4}$

D. $\frac{1}{12}(6\sqrt{6} - 2\sqrt{2})$

E. $\frac{1}{6}(6\sqrt{6} - 2\sqrt{2})$

11. Find the volume of the solid bounded by the surface $y = x^2$, $z = 0$, and $y + z = 1$.

A. $\frac{3}{5}$

B. $\frac{4}{15}$

C. $\frac{8}{15}$

D. $\frac{2}{3}$

E. $\frac{2}{5}$

12. Evaluate $\iiint_E z\,dV$ where E lies between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ in the first octant.

A. $\frac{7\pi}{12}$

B. $\frac{7\pi}{6}$

C. $\frac{7}{12}$

D. $\frac{15\pi}{8}$

E. $\frac{15\pi}{16}$