NAME ______________________________

STUDENT ID # ______________________________

INSTRUCTOR ______________________________

INSTRUCTIONS

1. There are 6 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. I.D.# is your 9 digit ID (probably your social security number). Also write your name at the top of pages 2–6.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. You need to show your work. Circle your answers in this test booklet for the first 10 questions.
4. No books, notes or calculators may be used on this exam.
5. Each problem is worth 10 points. The maximum possible score is 100 points.
6. Using a #2 pencil, fill in each of the following items on your answer sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION, write in your division and section number and fill in the little circles. (For example, for division 9 section 1, write 0901. For example, for division 38 section 2, write 3802).
   (c) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your student ID number, and fill in the little circles.
   (d) Using a #2 pencil, put your answers to questions 1–10 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
   (e) Sign your answer sheet.
7. After you have finished the exam, hand in your answer sheet and your test booklet to your instructor.

1. Find the rate of change of elevation in the direction of the steepest ascent for the surface $z = 2 - x^2 - y^4$ at the point $(1, 1, 0)$.

A. 1

B. $\sqrt{3}$

C. $\dfrac{\sqrt{3}}{3}$

D. $2\sqrt{5}$

E. $\sqrt{8}$

2. The function $f(x, y) = x^2 - 4xy + 8y^3 - 1$ has

A. a relative max value and a saddle point

B. a relative min value and a saddle point

C. an absolute minimum value

D. two relative maximum values

E. two relative minimum values

3. Find the largest area of a rectangle inscribed in the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

A. 6

B. 8

C. 9

D. 12

E. 16

4. Find a point on the unit sphere $x^2 + y^2 + z^2 = 1$ where the tangent plane is parallel to the plane $x + 2y + 2z = 10$.

A. $(1, 0, 0)$

B. $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}\right)$

C. $\left(\frac{1}{3}, \frac{1}{3}, \frac{\sqrt{7}}{3}\right)$

D. $\left(\frac{2}{3}, 0, \frac{1}{3}\right)$

E. $\left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$

5. The volume of the solid region lying under the hyperbolic paraboloid $z = 3y^2 - 3x^2$ and above the rectangle $[-1, 1] \times [1, 3]$ is:

A. 42

B. 46

C. 48

D. 52

E. 56

6. After reversing the order of integration, $\int_0^2 \int_{x^2}^4 x^3 \sin(y^3) dy dx$ is equal to:

A. $\int_0^4 \int_{\sqrt{y}}^2 x^3 \sin(y^3) dx dy$

B. $\int_0^4 \int_0^{\sqrt{y}} x^3 \sin(y^3) dx dy$

C. $\int_0^2 \int_{\sqrt{y}}^4 x^3 \sin(y^3) dx dy$

D. $\int_0^2 \int_4^{y^2} x^3 \sin(y^3) dx dy$

E. $\int_0^2 \int_0^{y^2} x^3 \sin(y^3) dx dy$

7. If D is the region bounded by $x = 0,\ x = 1,\ y = 0$, and $y = \sqrt{x}$, then $\displaystyle\iint_D \frac{2y}{x^2+1}\,dA$

is:

A. 2

B. $\arctan 2$

C. $\arctan\sqrt{\pi}$

D. $\dfrac{1}{2}\ln 2$

E. π

8. Express $\displaystyle\iint_R (x^2+2y^2)dxdy$, where R is the region $x^2+y^2 \le 1$, as an iterated integral in polar coordinates

A. $\displaystyle\int_0^{2\pi}\int_0^1 r^3(1+\sin^2\theta)drd\theta$

B. $\displaystyle\int_0^{2\pi}\int_0^1 r^2(1+\sin^2\theta)drd\theta$

C. $\displaystyle\int_0^{2\pi}\int_0^1 r(r+\sin^2\theta)drd\theta$

D. $\displaystyle\int_0^{1}\int_0^{2\pi} r(1+\sin^2\theta)d\theta dr$

E. $\displaystyle\int_0^{1}\int_0^{2\pi} (1+\sin^2\theta)d\theta dr$

9. The region D bounded by $y = x$ and $y = x^2$ has area 1/6. Let $(\overline{x}, \overline{y})$ be the center of mass of a homogeneous lamina in the shape of the region D. Find $\overline{y}$.

A. $\frac{6}{15}$

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $\frac{6}{13}$

E. $\frac{6}{11}$

10. Find the surface area of the part of the paraboloid $z = \frac{x^2}{2} + \frac{y^2}{2} + 3$ that lies inside the cylinder $x^2 + y^2 = 1$.

A. $\frac{2\pi}{3}(\sqrt{8}-1)$

B. $\pi(\sqrt{8}-3)$

C. $\frac{3\pi}{2}$

D. $\frac{\pi}{3}(\sqrt{8}-2)$

E. $\frac{2\pi}{3}(\sqrt{2}-1)$