NAME ______________________________

STUDENT ID # ______________________________

INSTRUCTOR ______________________________

INSTRUCTIONS

1. There are 6 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. I.D.# is your 9 digit ID (probably your social security number). Also write your name at the top of pages 2–6.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet for the first 7 questions. Partial credit will be given for work on the last 3 questions. You need to show your work.
4. No books, notes or calculators may be used on this exam.
5. Each problem is worth 10 points. The maximum possible score is 100 points.
6. Using a #2 pencil, fill in each of the following items on your answer sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION, write in your division and section number and fill in the little circles. (For example, for division 9 section 1, write 0901. For example, for division 38 section 2, write 3802).
   (c) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your student ID number, and fill in the little circles.
   (d) Using a #2 pencil, put your answers to questions 1–7 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
   (e) Sign your answer sheet.
7. After you have finished the exam, hand in your answer sheet and your test booklet to your instructor.

1. Let $z = f(x, y)$ where f is a differentiable function, $x = t^3 - 1$ and $y = t^2 + 2t + 1$. If $\frac{dz}{dt} = 4$ when $t = 0$, find $\frac{\partial z}{\partial y}$ when $(x, y) = (-1, 1)$.

A. 0

B. 2

C. -2

D. $\frac{3}{2}$

E. 4

2. Let $f(x, y) = x^3 y$. Find the value of the directional derivative at $(1, 2)$ in the direction in which f decreases most rapidly.

A. $-\sqrt{52}$

B. -1

C. $-\sqrt{37}$

D. 0

E. $-\sqrt{7}$

3. Find the equation of the tangent plane to the graph of the function $f(x,y) = (x^2+y^2)^2$ at the point $(1,1,4)$.

A. $(x-8)+(y-8)+4(z+1)=0$

B. $2x-2y+4z=16$

C. $x+y-8z=6$

D. $4x+4y-2z=0$

E. $8x+8y-z=12$

4. The extreme values of $f(x,y) = 4x - y + 2$ subject to the constraint $2x^2 + y^2 = 1$ are

A. maximum 5, minimum -1

B. maximum 4, minimum 2

C. maximum 4, no minimum

D. no maximum, minimum -1

E. maximum 4, minimum -1

5. Change the order of integration and evaluate

$$\int_0^1 \int_{\sqrt{x}}^1 e^{y^3}\, dy\, dx.$$

A. $\frac{1}{2}e$
B. $\frac{1}{2}(e-1)$
C. $\frac{1}{3}e$
D. $\frac{1}{3}(e-1)$
E. e

6. An object occupies the region in the first octant bounded by the coordinate planes and by the paraboloid $z = 4 - x^2 - y^2$. The mass density at a given point in the object is equal to the distance from the xy-plane. The total mass m of the object is given by the triple integral

A. $\displaystyle\int_0^2 \int_0^{4-x^2} \int_0^{4-x^2-y^2} z^2\, dz\, dy\, dx$

B. $\displaystyle\int_0^2 \int_0^{\sqrt{4-x^2}} \int_0^{4-x^2-y^2} z\, dz\, dy\, dx$

C. $\displaystyle\int_0^2 \int_0^{4-x^2} \int_0^{4-x^2-y^2} z\, dz\, dy\, dx$

D. $\displaystyle\int_0^4 \int_0^{\sqrt{4-x^2}} \int_0^{4-x^2-y^2} z\, dz\, dy\, dx$

E. $\displaystyle\int_0^2 \int_0^{\sqrt{4-y^2}} \int_0^{4-x^2-y^2} z^2\, dz\, dx\, dy$

7. Find the volume of the solid region bounded below by the surface $z = \sqrt{r}$ (in cylindrical coordinates) and above by the plane $z = 1$.

A. $\frac{\pi}{4}$

B. $\frac{\pi}{3}$

C. $\frac{2\pi}{7}$

D. $\frac{\pi}{6}$

E. $\frac{\pi}{5}$

8. Evaluate the integral

$$\iint_R \cos(x^2 + y^2) dA$$

where R is the region in the xy-plane bounded by the circle $x^2 + y^2 = \frac{\pi}{2}$.

9. Find the surface area of that part of the paraboloid $z = x^2 + y^2$ that is below the plane $z = 4$ and above the plane $z = 1$.

10. Set up a triple iterated integral in rectangular coordinates which gives the volume of the region bounded above by the surface $z = 5 - x^2$ and below by the surface $z = 2 + x^2 + y^2$. Do not evaluate the integral!