MA 26100
EXAM 1 Green
October 4, 2017

NAME _____ YOUR TA'S NAME _____

STUDENT ID # _____ RECITATION TIME _____

1. You must use a #2 pencil on the mark–sense sheet (answer sheet).

2. Be sure the paper you are looking at right now is GREEN!

3. Write the following in the TEST/QUIZ NUMBER boxes (and blacken in the appropriate spaces below the boxes):

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4. On the mark-sense sheet, fill in your TA's name and the course number.

5. Fill in your NAME and STUDENT IDENTIFICATION NUMBER and blacken in the appropriate spaces.

6. Fill in your four-digit SECTION NUMBER. If you do not know your section number, please ask your TA.

7. Sign the mark–sense sheet.

8. Fill in your name, etc. on this paper (above).

9. There are 12 questions, each worth 8 points (you will automatically earn 4 points for taking the exam). Blacken in your choice of the correct answer in the spaces provided for questions 1–12. Do all your work on the question sheets.

10. Turn in both the mark–sense sheets and the question sheets when you are finished.

11. If you finish the exam before 8:50, you may leave the room after turning in the scantron sheet and the exam booklet. You may not leave the room before 8:20.
If you don't finish before 8:50, you MUST REMAIN SEATED until your TA comes and collects your scantron sheet and your exam booklet.

12. NO CALCULATORS, PHONES, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

EXAM POLICIES

1. Students may not open the exam until instructed to do so.
2. Students must obey the orders and requests by all proctors, TAs, and lecturers.
3. No student may leave in the first 20 min or in the last 10 min of the exam.
4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

1. Suppose that the three points $(0,0,0)$, $(1,1,1)$, and (a,b,c) lie in a unique plane P. The line passing through $(0,0,0)$ and $(1,2,a)$ is normal to P. What is a?

A. $a = -3$

B. $a = 0$

C. $a = 1$

D. $a = 2$

E. $a = -1$

2. Suppose a particle has acceleration

$$\mathbf{a}(t) = \frac{-1}{(1+t)^2}\mathbf{i} + \sqrt{t}\mathbf{j} + 2\mathbf{k}.$$

If the inital velocity $\mathbf{v}(0) = \mathbf{i} + \mathbf{j} + \mathbf{k}$, and the initial position is $\mathbf{r}(0) = \mathbf{0}$, what is $\mathbf{r}(1)$?

A. $(\ln 2)\mathbf{i} + \frac{4}{15}\mathbf{j} + 2\mathbf{k}$

B. $(\ln 2)\mathbf{i} + \frac{19}{15}\mathbf{j} + 2\mathbf{k}$

C. $(1 + \ln 2)\mathbf{i} + \frac{4}{15}\mathbf{j}$

D. $(1 + \ln 2)\mathbf{i} + \frac{19}{15}\mathbf{j} + 2\mathbf{k}$

E. $\mathbf{i} + \frac{4}{15}\mathbf{j}$

3. Consider the space curve

$$\mathbf{r}(t) \ = (\cos t + \sin t)\mathbf{i} \ + \ (\cos t - \sin t)\mathbf{j} \ + \ t\mathbf{k}.$$

Compute $\mathbf{N}(t)$, the principal unit normal vector.

A. $(-\sin t)\mathbf{i} + (\cos t)\mathbf{j}$

B. $\frac{1}{\sqrt{3}}(\cos t - \sin t)\mathbf{i} - \frac{1}{\sqrt{3}}(\cos t + \sin t)\mathbf{j} \ + \frac{1}{\sqrt{3}}\mathbf{k}$

C. $-\frac{1}{\sqrt{2}}(\cos t + \sin t)\mathbf{i} + \frac{1}{\sqrt{2}}(\sin t - \cos t)\mathbf{j}$

D. $\frac{1}{\sqrt{3}}(\cos t - \sin t)\mathbf{i} - \frac{1}{\sqrt{3}}(\cos t + \sin t)\mathbf{j}$

E. $\frac{1}{\sqrt{2}}(\cos t + \sin t)\mathbf{i} + \frac{1}{\sqrt{2}}(\sin t - \cos t)\mathbf{j}$

4. Consider the following two limits:

$$\text{I.} \lim_{(x,y)\to(0,0)} \frac{xy}{x^2+xy+y^2} \qquad \text{II.} \lim_{(x,y)\to(0,0)} \frac{x+y}{e^x+e^y}$$

A. I. does not exist, II. 0

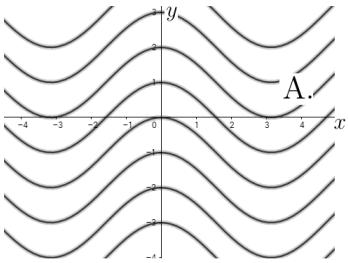
B. I. does not exist, II. does not exist

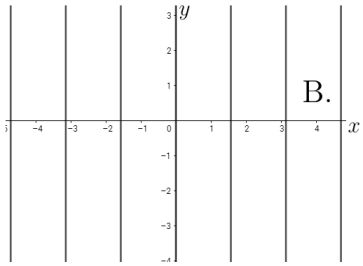
C. I. 0, II. does not exist

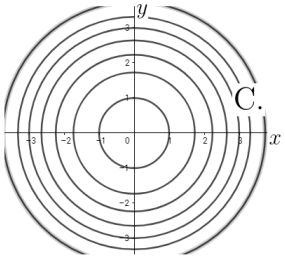
D. I. 0, II. 0

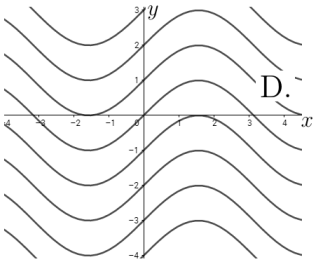
E. I. 0, II. 1

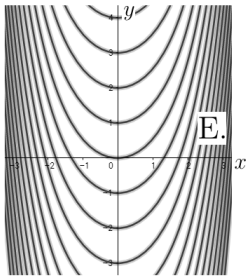
5. Suppose $z = f(x, y) = \cos x$. Choose the correct contour map (level curves of f):











6. Suppose the function $f(x, y)$ has the following properties:

$$f_x(0,0) = 11, \quad f_x(1,1) = 3, \quad f_y(0,0) = 5, \quad f_y(1,1) = 7.$$

If $x = e^s - te^t$ and $y = e^t - se^s$, then what is f_s at $(s, t) = (0, 0)$?

A. -2

B. 10

C. 16

D. 6

E. -4

7. The length and width of a rectangle are measured as 10 cm and 4 cm, respectively, with an error in measurement of at most 0.1 cm each. Use differentials to estimate the maximum error in the calculated area of the rectangle.

A. 5 cm^2

B. 1.4 cm^2

C. 0.5 cm^2

D. 4 cm^2

E. 0.4 cm^2

8. Find the point (x, y) at which f has a local minimum:

$$f(x, y) = \frac{1}{3}y^3 + x^2 + 4xy - 2x - 13y + 7.$$

A. $(1, -13)$

B. $(3, -1)$

C. $(1, -3)$

D. $(-17, 9)$

E. None of the above.

9. The depth of a lake at the point (x, y) is $3x^2y + 5y^2$ feet. Assume that x and y are measured in miles. If a boat at the point $(2, 1)$ is sailing in the direction of the vector $3\mathbf{i} + \mathbf{j}$, at what rate is the water depth changing?

A. 12 feet/mile

B. $14\frac{1}{2}$ feet/mile

C. $\dfrac{58}{\sqrt{10}}$ feet/mile

D. 41 feet/mile

E. $\dfrac{23\sqrt{10}}{5}$ feet/mile

10. Find $\dfrac{\partial z}{\partial x}$ at $(x, y, z) = (1, 2, 1)$, where z is defined implicitly as a function of x and y by the equation

$$\tan(\pi xyz) = x^2 - z^2.$$

A. $\dfrac{1}{\pi + 1}$

B. $1 - \pi$

C. $\dfrac{2 - 2\pi}{3}$

D. $\dfrac{1 - \pi}{1 + \pi}$

E. $\dfrac{\pi + 1}{1 - \pi}$

11. Find the tangent plane to the level surface $xy^2z^3 = 12$ at $(3, 2, 1)$.

A. $x + 2y + 3z = 10$

B. $x + y + z = 6$

C. $3x + 2y + z = 14$

D. $x + 3y + 6z = 15$

E. $x + 3y + 9z = 18$

12. Find $f'(1)$, where $f(t) = \mathbf{u}(t) \cdot \mathbf{v}(t)$, $\mathbf{u}(1) = \langle 1, 1, 1 \rangle$, $\mathbf{u}'(1) = \langle 1, 2, 3 \rangle$, and $\mathbf{v}(t) = \langle t, t^2, t^3 \rangle$.

A. 6

B. 14

C. 28

D. 12

E. 24