MA 261 | Exam 1 | September 22, 2005

Name: ______________________________

Student ID Number: ______________________________

Lecturer: ______________________________

Recitation Instructor: ______________________________

Instructions:

1. This exam contains 12 problems worth 8 points.
2. Please supply all information requested above and on the mark-sense sheet.
3. Work only in the space provided, or on the backside of the pages. Mark your answers clearly on the scantron. Also circle your choice for each problem in this booklet.
4. No books, notes, or calculator, please.

Key: DBEB CCAB DAEC

1. If $M = (0,3,1)$ and $N = (2,3,-1)$ then a unit vector in the direction of $\vec{MN}$ is:

A. $\langle 2,0,-2\rangle$

B. $\langle 2,0,0\rangle$

C. $\langle -2,0,2\rangle$

D. $\langle \frac{2}{\sqrt{8}},\ 0,\ -\frac{2}{\sqrt{8}}\rangle$

E. $\langle -\frac{2}{\sqrt{8}},\ 0,\ \frac{2}{\sqrt{8}}\rangle$

2. The angle between the vectors $3\vec{i} + \sqrt{3}\vec{k}$ and $\vec{k}$ is:

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{2}$

E. π

3. Which of the following statements is true for all three-dimensional vectors $\vec{a}, \vec{b}$, and $\vec{c}$, if θ is the angle between $\vec{a}$ and $\vec{b}$?

(i) $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$

(ii) $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{b} \times \vec{c}) \cdot \vec{a}$

(iii) $|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \cdot \cos\theta$

(iv) $(\vec{a} \times \vec{b}) \cdot a = 0$

A. All are true

B. (i), (ii), and (iii) only

C. (i), (ii), and (iv) only

D. (i), (iii) and (iv) only

E. (ii) and (iv) only

4. The equation $\rho = 2\cos\theta\sin\phi$ in spherical coordinates defines a sphere of radius $R =$

A. $\frac{1}{2}$

B. 1

C. $\frac{1}{4}$

D. 2

E. 4

5. Find the equation of the plane that contains the points $(1, 2, 1)$, $(2, -1, 0)$ and $(3, 3, 1)$.

A. $-x - 2y + 9z = 4$

B. $x + 2y + 7z = 12$

C. $x - 2y + 7z = 4$

D. $x + 2y + z = 6$

E. $-x + 2y + 9z = 12$

6. Find a vector function $\vec{r}(t)$, $0 \leq t \leq 2\pi$, that represents the curve of intersection of the cylinder $4x^2 + 4y^2 = 1$ with the plane $x + y + z = 2$.

A. $\vec{r}(t) = 2\cos t\vec{i} + 2\sin t\vec{j} + (2 - 2\cos t - 2\sin t)\vec{k}$

B. $\vec{r}(t) = 2\cos t\vec{i} + 2\sin t\vec{j} + (2 + 2\cos t + 2\sin t)\vec{k}$

C. $\vec{r}(t) = \frac{1}{2}\cos t\vec{i} + \frac{1}{2}\sin t\vec{j} + (2 - \frac{1}{2}\cos t - \frac{1}{2}\sin t)\vec{k}$

D. $\vec{r}(t) = \frac{1}{2}\cos t\vec{i} + \frac{1}{2}\sin t\vec{j} + (2 + \frac{1}{2}\cos t + \frac{1}{2}\sin t)\vec{k}$

E. $\vec{r}(t) = 2\cos t\vec{i} + 2\sin t\vec{j} + (2 - \cos t - \sin t)\vec{k}$

7. Find parametric equations for the tangent line to the curve

$$\vec{r}(t) = \langle t^2 + 3t + 2,\ e^t \cos t,\ \ln(t+1) \rangle$$

at the point $(2, 1, 0)$.

A. $x = 2 + 3t \quad y = 1 + t \quad z = t$

B. $x = 2t+3,\ y = e^t(\cos t - \sin t),\ z = \dfrac{1}{t+1}$

C. $x = 3 + 2t \quad y = 1 + t \quad z = 1$

D. $x = 3t \quad y = 2t \quad z = 1 + t$

E. $x = 2 - t \quad y = 1 + t \quad z = 3 - 3t$

8. Find the arc–length of the curve defined by $\vec{r}(t) = \langle t,\ \frac{\sqrt{6}}{2}\, t^2,\ t^3 \rangle,\ -1 \leq t \leq 1.$

A. 5

B. 4

C. 3

D. 2

E. 6

9. A particle starts at the origin with initial velocity $\vec{\imath}+\vec{\jmath}-\vec{k}$. Its acceleration is $\vec{a}(t) = t\vec{\imath}+\vec{\jmath}+t\vec{k}$. Find its position at $t=1$.

A. $\frac{1}{6}\,\vec{\imath}+\frac{1}{2}\,\vec{\jmath}+\frac{1}{3}\,\vec{k}$

B. $\frac{7}{6}\,\vec{\imath}+\frac{1}{2}\,\vec{\jmath}-\frac{5}{6}\,\vec{k}$

C. $\vec{\imath}+\vec{\jmath}+\vec{k}$

D. $\frac{7}{6}\,\vec{\imath}+\frac{3}{2}\,\vec{\jmath}-\frac{5}{6}\,\vec{k}$

E. $\vec{\imath}+2\vec{\jmath}-\vec{k}$

10. The level surface of $f(x,y,z)=x^2+y^2-z^2$ corresponding to $f(x,y,z)\equiv 1$ intersects the xy-plane in a

A. circle

B. parabola

C. ellipse

D. hyperbola

E. line

11. If $L = \lim_{(x,y,z)\to(0,0,0)} \dfrac{x+2y-3z}{\sqrt{x^2+y^2+z^2}}$, then

A. $L = 1$

B. $L = -2$

C. $L = -3$

D. $L = 0$

E. the limit does not exist

12. If $f(x,y) = \ln(x^2+2y^2)$, then the partial derivative f_{xy} equals

A. $\dfrac{4xy}{(x^2+2y^2)^2}$

B. $\dfrac{4(x^2-y^2)}{(x^2+2y^2)^2}$

C. $\dfrac{-8xy}{(x^2+2y^2)^2}$

D. $\dfrac{-4y}{(x^2+2y^2)^2}$

E. $\dfrac{-2x}{(x^2+2y^2)^2}$