MA 16600
FINAL EXAM VERSION 01
May 3, 2023

INSTRUCTIONS

Your name ____________________ Your TA's name ____________________

Student ID # ________________ Section # and recitation time ____________

**1.** You must use a #2 pencil on the scantron sheet (answer sheet).

**2.** Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

**3.** On the scantron sheet, fill in your **TA's name, i.e., the name of your recitation instructor (NOT the lecturer's name)** and the course number.

**4.** Fill in your **NAME** and **PURDUE ID NUMBER**, and blacken in the appropriate spaces. Put 00 at the front of PUID to make it a 10 digit number, and then fill it in.

**5.** Fill in the four-digit **SECTION NUMBER**. Your section number is a 3 digit number. Put 0 at the front to make it a 4 digit number, and then fill it in.

**6. Sign the scantron sheet.**

**7.** Blacken your choice of the correct answer in the space provided for each of the questions 1–25. While mark all your answers on the scantron sheet, you should **show your work** on the exam booklet. Although no partial credit will be given, any disputes about the grade or grading will be settled by examining your written work on the exam booklet.

**8.** There are 25 questions, each of which is worth 8 points. The maximum possible score is

$25 \text{ questions } \times 8 \text{ points } = 200 \text{ points}.$

**9.** **NO calculators, electronic device, books, or papers are allowed.** Use the back of the test pages for scrap paper.

**10.** After you finish the exam, **turn in BOTH the scantron sheet and the exam booklet**.

**11.** If you finish the exam before 5:25 PM, you may leave the room after turning in the scantron sheet and the exam booklet. **If you don't finish before 5:25 PM, you should REMAIN SEATED** until your TA comes and collects your scantron sheet and exam booklet.

# Exam Policies

1. There is no individual seating. Just follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs/proctors will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

# Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor/proctor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ________________________________________

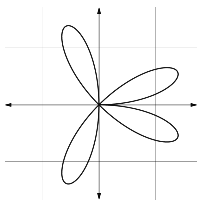
STUDENT SIGNATURE: ____________________________________

## Questions

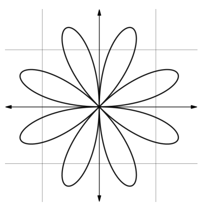
**1.** (8 points) Choose the picture which best depicts the curve defined by the following equation in polar coordinates

$$r = 3\sin(4\theta), \quad 0 \leq \theta \leq \pi.$$

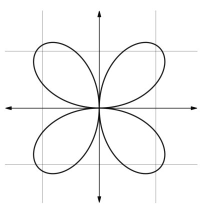
A.



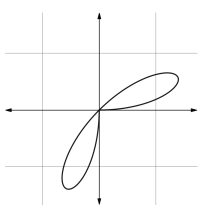
B.



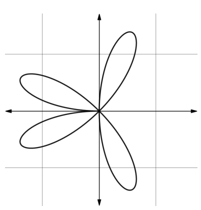
C.



D.



E.



**2.** (8 points) Consider the following two complex numbers

$$z = 4 + 4\sqrt{3}i$$
$$w = \sqrt{3} + i$$

then $\dfrac{z}{w}$ in polar coordinates is given by:

A. $4\left\{\cos\left(\frac{\pi}{3} - \frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{3} - \frac{\pi}{6}\right)\right\}$

B. $4\left\{\cos\left(\frac{\pi}{3} + \frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)\right\}$

C. $4\left\{\cos\left(\frac{\pi}{3} \cdot \frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{3} \cdot \frac{\pi}{6}\right)\right\}$

D. $8\left\{\cos\left(\frac{\pi}{3} - \frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{3} - \frac{\pi}{6}\right)\right\}$

E. $8\left\{\cos\left(\frac{\pi}{3} / \frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{3} / \frac{\pi}{6}\right)\right\}$

**3.** (8 points) The graph of the polar equation $r = -8\sin\theta$ is

A. a vertical line

B. a horizontal line

C. a circle of radius 4 centered at $(0, -4)$

D. a circle of radius 4 centered at $(0, 4)$

E. a circle of radius 4 centered at $(-4, 0)$

**4.** (8 points) Find the area between the curves defined by the polar equations

$$r = e^{\frac{\theta}{2}} \;\; \& \;\; r = 1 \quad 0 \leq \theta \leq 1.$$

A. $\dfrac{e-2}{2}$

B. $e - 2$

C. $\dfrac{2-e}{2}$

D. $e^{1/2} + \dfrac{1}{2}$

E. $e - 1$

**5.** (8 points) The equation of a plane is given by:

$$3x - 8y + 4z = 17.$$

Which of the following vectors is parallel to the plane ?

A. $\langle 1, 2, -1 \rangle$

B. $\langle 3, -8, 4 \rangle$

C. $\langle 0, 1, 2 \rangle$

D. $\langle 2, 1, 0 \rangle$

E. None of the vectors above is parallel to the plane.

**6.** (8 points) Find the volume of the parallelopiped, where the following three vectors correspond to the three sides of the parallelopiped are given by

$$\begin{cases} \overrightarrow{OA} &= \langle 1, 1, 1 \rangle \\ \overrightarrow{OB} &= \langle 0, -1, 1 \rangle \\ \overrightarrow{OC} &= \langle 1, -1, 0 \rangle \end{cases}$$

A. 1

B. 2

C. 3

D. 4

E. 5

**7.** (8 points) Find the area of the region bounded by the curves $x = 3 + y^2$ and $x = 7$.

A. $\frac{16}{9}$

B. $\frac{16}{3}$

C. $\frac{32}{9}$

D. 16

E. $\frac{32}{3}$

**8.** (8 points) If the work required to stretch a spring 2 ft beyond its natural length is 18 ft-lb, how much work is needed to stretch it 4 inches beyond natural length ?

A. $\frac{81}{2}$ ft-lb

B. $\frac{9}{2}$ ft-lb

C. $\frac{1}{2}$ ft-lb

D. 81 ft-lb

E. $\frac{27}{2}$ft-lb

**9.** (8 points) Find the volume of the solid generated by revolving about the y-axis the region bounded by

$$y = 0, \quad y = \frac{x}{1+x^3}, \quad x = 0, \text{ and } x = 2.$$

A. $\dfrac{2\pi}{3} \arctan(9)$

B. $2\pi \ln(9)$

C. $\pi \ln(9)$

D. $\dfrac{2\pi}{3} \ln(9)$

E. $\pi \arctan(9)$

**10.** (8 points) The graph of $f(x) = \dfrac{e^x + e^{-x}}{2}$ on the interval $[-2, 2]$ is revolved about the $x$-axis.

Find the right formula that describes the area of the surface generated.

A. $\displaystyle\int_{-2}^{2} \pi \left(e^{2x} - e^{-2x}\right)\, dx$

B. $\displaystyle\int_{-2}^{2} 2\pi \left(\frac{e^x + e^{-x}}{2}\right)\left(\frac{e^x - e^{-x}}{2}\right) dx$

C. $\displaystyle\int_{-2}^{2} 2\pi \left(\frac{e^x - e^{-x}}{2}\right)^2 dx$

D. $\displaystyle\int_{-2}^{2} 2\pi \left(\frac{e^x + e^{-x}}{2}\right)^2 dx$

E. $\displaystyle\int_{-2}^{2} 2\pi \left(\frac{e^x + e^{-x}}{2}\right)\sqrt{1 + \frac{e^{2x}}{4} + \frac{e^{-2x}}{4}}\, dx$

**11.** (8 points) Compute $\int_0^1 x^2 e^x dx$.

A. $e - 2$

B. $e$

C. $2e$

D. $e + 2$

E. $1$

**12.** (8 points) Write out the form of the partial fraction decomposition of the function

$$\frac{5x^3 + 8x^2 - 21}{(x-1)^2(x^2+9)(x^3-1)}.$$

A. $\dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x-3} + \dfrac{D}{x+3} + \dfrac{Ex+F}{x^3-1}$

B. $\dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{(x-1)^3} + \dfrac{D}{x+3} + \dfrac{E}{x-3} + \dfrac{Fx+G}{x^2+x+1}$

C. $\dfrac{Ax+B}{(x-2)^2} + \dfrac{Cx+D}{x^2+9} + \dfrac{Ex^2+Fx+G}{x^3-1}$

D. $\dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{Cx+D}{x^2+9} + \dfrac{Ex+F}{x^2+x+1}$

E. $\dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{(x-1)^3} + \dfrac{Dx+E}{x^2+9} + \dfrac{Fx+G}{x^2+x+1}$

Note: The letters $A, B, C, D, E, F, G$ in the partial fractions above represent some appropriate constants.

**13.** (8 points) Compute the integral

$$\int \frac{1}{x^2+4x+3}\,dx.$$

A. $\ln|x+1| - \ln|x+3| + C$

B. $\ln|x^2+3x+2| + C$

C. $\dfrac{1}{2}\ln|x+1| - \dfrac{1}{2}\ln|x+3| + C$

D. $\ln|x+3| - \ln|x+1| + C$

E. $\tan^{-1}(x+2) + C$

**14.** (8 points) Evaluate the improper integral $\int_0^\infty 2x^3 e^{-x^2}\, dx$.

A. 0

B. 1

C. 2

D. $\dfrac{1}{2}$

E. $\infty$

**15.** (8 points) Compute $\int_0^{\frac{\pi}{2}} \sin^5 x \; dx$.

A. $\frac{8}{15}$

B. $\frac{-8}{15}$

C. $\frac{7}{15}$

D. $\frac{-7}{15}$

E. $\frac{2}{3}$

**16.** (8 points) Compute the integral $\int_0^{\frac{\pi}{4}} \tan(x)\sec^3(x)\ dx$.

A. $\dfrac{4\sqrt{2}-1}{5}$

B. $\dfrac{4\sqrt{2}}{5}$

C. $\dfrac{2\sqrt{2}-1}{3}$

D. $\dfrac{2\sqrt{2}}{3}$

E. $\dfrac{3}{4}$

**17.** The appropriate trigonometric substitution will convert the following integral

$$\int_2^7 \sqrt{x^2 - 4x + 29}\; dx$$

into:

A. $25 \int_0^{\frac{\pi}{4}} \sec^3\theta \; d\theta$

B. $5 \int_0^{\frac{\pi}{4}} \sec^3\theta \; d\theta$

C. $25 \int_0^{\frac{\pi}{2}} \sec\theta \tan^2\theta \; d\theta$

D. $5 \int_0^{\frac{\pi}{2}} \sec\theta \tan^2\theta \; d\theta$

E. $25 \int_0^{\frac{\pi}{2}} \sec^2\theta \tan\theta \; d\theta$

**18.** (8 points) Evaluate the integral

$$\int_4^{11/2} \frac{1}{\sqrt{-x^2+8x-7}}\, dx.$$

A. $\frac{\pi}{2}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$

D. $\frac{\sqrt{2}}{2}$

E. $\frac{\sqrt{3}}{2}$

**19.** (8 points) Which of the following series converges absolutely ?

A. $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^k}{\sqrt[3]{k^3+k}}$

B. $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^k}{k^3+k}$

C. $\displaystyle\sum_{k=2}^{\infty} (-1)^k \sqrt[k]{\frac{1}{k^5}}$

D. $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^k}{\ln k}$

E. $\displaystyle\sum_{k=2}^{\infty} \frac{(-7)^k}{k^2}$

**20.** (8 points) Find the interval of convergence for the power series

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{4^k(k+5)}(x-3)^{2k}.$$

A. $[1, 5)$

B. $(1, 5]$

C. $[1, 5]$

D. $(-1, 7]$

E. $[-1, 7)$

**21.** Compute the value of the series

$$\sum_{k=0}^{\infty} \frac{k^2 + 3k + 2}{3^k}$$

HINT:

(1) Observe

$$\sum_{k=0}^{\infty} (k^2 + 3k + 2)x^k = \sum_{k=0}^{\infty} (k+2)(k+1)x^k$$

(2) Starting from the power series

$$f(x) = \frac{1}{1-x} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + x^3 + x^4 + \cdots ,$$

compute the power series for $f''(x)$. After an appropriate index change for $k$, how does the power series for $f''(x)$ compare with the one in (1) ?

(3) What value should one put for $x$ to evaluate the given series ?

A. 8

B. 27/4

C. 27/8

D. 3/4

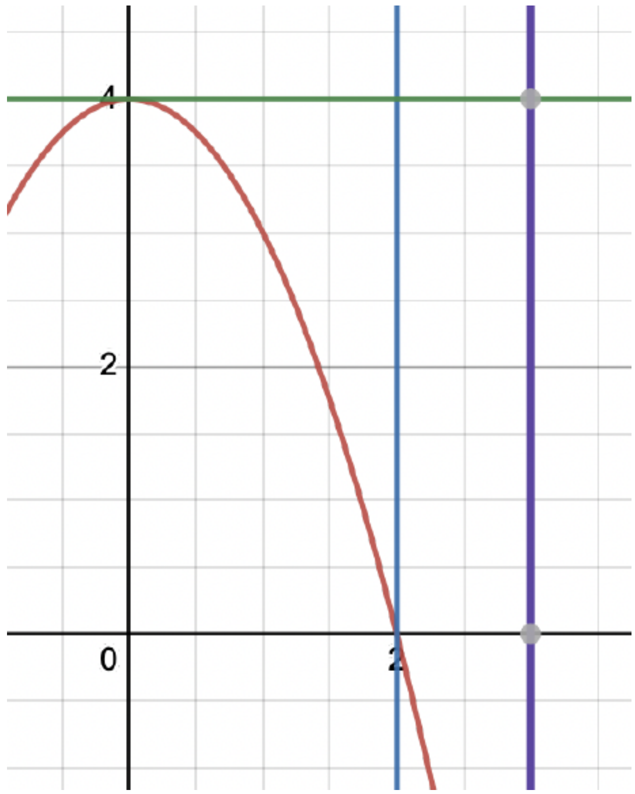
E. $2 \ln 2$

**22.** (8 points) Choose the right formulas to compute the volume of the solid obtained from rotating **about the line** $x = 3$ the region given below, using

(i) Washer method,

(ii) Shell method.

The region is bounded by $y = -x^2 + 4, x = 2$ and $y = 4$.




A. (i) $\int_0^4 \pi \left\{(3 - \sqrt{4-y})^2 - 1^2\right\} dy$, (ii) $\int_0^2 2\pi(3-x)\left\{4 - (-x^2+4)\right\} dx$

B. (i) $\int_0^4 \pi \left(5 - \sqrt{4-y}\right)^2 dy$, (ii) $\int_0^2 2\pi x(-x^2+4)\ dx$

C. (i) $\int_0^2 \pi \left(4 - (-x^2+4)\right)^2 dx$, (ii) $\int_0^4 2\pi(3-y)\left\{2 - \sqrt{4-y}\right\} dy$

D. (i) $\int_0^2 \pi \left\{4^2 - (-x^2+4)^2\right\} dx$, (ii) $\int_0^4 2\pi y\left(2 - \sqrt{4-y}\right)\ dy$

E. (i) $\int_0^4 \pi \left\{1^2 - (3 - \sqrt{4-y})^2\right\} dy$, (ii) $\int_0^2 2\pi(x-3)\left\{4 - (-x^2+4)\right\} dx$

**23.** (8 points) The Alternating Series Test shows that

$$\sum_{k=1}^{\infty} (-1)^k \frac{1}{\sqrt{7k}}$$

converges to a value $S$.

Set

$$S_N = \sum_{k=1}^{N} (-1)^k \frac{1}{\sqrt{7k}}.$$

Find the smallest $N$, using The Estimation Theorem for Alternating Series, such that we can conclude

$$|S - S_N| < \frac{1}{10^2}.$$

A. 1427

B. 1428

C. 1429

D. 100

E. 101

**24.** (8 points) Given that $\displaystyle\sum_{k=0}^{\infty} \frac{5^k}{(k+3)!}(x-2)^{2k}$ is the Taylor series for the function $f(x)$ centered at $x=2$.

The value of $f^{(5)}(2)$ is:

A. $\dfrac{4!\,5^6}{8!}$

B. $\dfrac{5!\,5^6}{8!}$

C. $\dfrac{4!\,5^6}{7!}$

D. $\dfrac{6!\,5^6}{8!}$

E. 0

**25.** (8 points) Consider the series

$$\sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} \left( \frac{k}{k^2 + 2k + 3} \right) \sin\left(\frac{1}{k}\right).$$

We conclude

A. the series is convergent since $\lim_{k\to\infty} a_k = 0$.

B. the series is divergent by Ratio Test since $\lim_{k\to\infty} \left| \frac{a_{k+1}}{a_k} \right| = 1$.

C. the series is divergent by Limit Comparison Test with $\sum_{k=1}^{\infty} \frac{k}{k^2} = \sum_{k=1}^{\infty} \frac{1}{k}$.

D. the series is convergent by Limit Comparison Test with $\sum_{k=1}^{\infty} \frac{1}{k^2}$.

E. the series is divergent by Test for Divergence since $\lim_{k\to\infty} a_k$ does not exist.