MA 16600
FINAL EXAM INSTRUCTIONS
VERSION 01
May 5, 2016

Your name ____________________ Your TA's name ____________________

Student ID # _______________ Section # and recitation time __________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your TA's name (NOT the lecturer's name) and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit SECTION NUMBER.

6. Sign the scantron sheet.

7. Write down YOUR NAME and TA's NAME on the exam booklet.

8. There are 20 questions, each worth 10 points. Blacken your choice of the correct answer in the spaces provided for questions 1–20. Do all your work on the question sheets. Turn in both the scantron sheets and the question sheets when you are finished.

9. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

10. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

11. After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

12. If you finish the exam before 2:55, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 2:55, you should REMAIN SEATED until your TA comes and collects your scantron sheets and exam booklets.

Questions

1. For what value of t is the vector $\vec{a} = \langle 2, t, -1 \rangle$ perpendicular to the vector $\vec{b} = \langle t, 1, 1 \rangle$?

A. $t = \dfrac{1}{5}$

B. $t = \dfrac{1}{3}$

C. $t = \dfrac{2}{3}$

D. $t = -1$

E. $t = -2$.

2. Which of the following statements are true for any three-dimensional vectors $\vec{a}$ and $\vec{b}$?

(I) $(\vec{a} \times \vec{b}) \cdot \vec{a} = 0$
(II) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
(III) $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$
(IV) $|\vec{a} \cdot \vec{b}| \leq |\vec{a}|\,|\vec{b}|$

A. (II) and (III) only

B. (I), (II) and (III) only

C. (I), (II) and (IV) only

D. (II), (III) and (IV) only

E. All true

3. Find the value of c for which $x^2 + y^2 + z^2 - 2x + 4z = c$ defines a sphere of radius 3.

A. 0

B. 1

C. 2

D. 3

E. 4

4. $\displaystyle\int_0^{\pi/2} x \sin x \, dx =$

A. $\frac{1}{3}$

B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

E. 1

5. $\displaystyle\int_0^{\pi/2} \sin^3 x \cos^2 x \, dx =$

A. $\frac{1}{15}$

B. $\frac{2}{15}$

C. $\frac{1}{5}$

D. $\frac{1}{3}$

E. $\frac{1}{2}$

6. Use a trigonometric substitution to compute the integral $\displaystyle\int_0^1 \frac{x^2\,dx}{\sqrt{1-x^2}}$.

A. $\frac{\pi}{4}$

B. π

C. $\frac{\pi}{2}$

D. $\frac{2\pi}{3}$

E. $\frac{\pi}{3}$

7. The integral $\displaystyle\int \frac{x-1}{x^3+x}\,dx$ is of the form (where a, b, c are some constants):

A. $a\ \ln|x^3+x| + C$

B. $a\ \ln|x| + b\ \ln|x+1| + C$

C. $a\ \ln|x| + b\ \ln|x+1| + c\ \ln|x-1| + C$

D. $a\ \ln|x| + b\ \ln(x^2+1) + C$

E. $a\ \ln|x| + b\ \ln(x^2+1) + c\ \tan^{-1} x + C$

8. The region bounded by the graph of $y = 1 + x^2$ and the line $y = 2$ is rotated about the x-axis to form a solid. The volume of that solid is given by

A. $\displaystyle\int_1^2 2\pi[2^2 - (1 + x^2)^2]\, dx$

B. $\displaystyle\int_1^2 \pi[2^2 - (1 + x^2)^2]\, dx$

C. $\displaystyle\int_{-1}^1 2\pi[2^2 - (1 + x^2)^2]\, dx$

D. $\displaystyle\int_{-1}^1 \pi[2^2 - (1 + x^2)^2]\, dx$

E. $\displaystyle\int_1^2 \pi(1 - x^2)^2\, dx$

9. Which of these improper integrals converge?

(I) $\displaystyle\int_0^\infty \cos x\,dx$, (II) $\displaystyle\int_0^\infty \frac{x}{1+x^2}\,dx$, (III) $\displaystyle\int_0^1 \frac{1}{x}\,dx$

A. Only (I)

B. Only (II)

C. Only (III)

D. All of them

E. None of them

10. The length of the curve $y = \dfrac{x^2}{2} - \dfrac{\ln x}{4}, \quad 2 \le x \le 4,$ is

A. $\dfrac{\ln 2}{4}$

B. $2 + \dfrac{\ln 2}{4}$

C. $4 + \dfrac{\ln 2}{4}$

D. $6 + \dfrac{\ln 2}{4}$

E. $8 + \dfrac{\ln 2}{4}$

11. Find the y-coordinate of the centroid of the region in the first quadrant bounded by $x = 0$, $y = 0$, and $2y = \sqrt{2 - x^2}$. You may assume that the area of the region is $\pi/4$.

A. $\dfrac{2\sqrt{2}}{3\pi}$

B. $\dfrac{\sqrt{2}}{3\pi}$

C. $\dfrac{\sqrt{2}}{2\pi}$

D. $\dfrac{\sqrt{2}}{4\pi}$

E. $\dfrac{4\sqrt{2}}{3\pi}$

12. Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{3 \cdot 2^n + (-3)^{n+1}}{5^n}$.

A. $\dfrac{25}{2}$

B. $\dfrac{25}{6}$

C. $\dfrac{25}{4}$

D. $\dfrac{25}{8}$

E. $\dfrac{-25}{4}$

13. Assume that $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$. According to the Alternating Series Test what is the smallest number of terms of the series that one needs to add to compute $\ln(1.1)$ with error less than or equal to 10^{-8}.

A. 10

B. 8

C. 9

D. 7

E. 11

14. Test the following series for convergence:

(I) $\sum_{n=1}^{\infty} 5^{-n}$

(II) $\sum_{n=1}^{\infty} \sin \frac{n\pi}{2}$

(III) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

A. I and II are convergent and III is divergent.

B. II and III are convergent and I is divergent.

C. I and III are convergent and II is divergent.

D. I, II, and III are convergent.

E. I, II, III are divergent.

15. Determine which of the following sequences converge.

(I) $a_n = \dfrac{n!}{(n+1)!}$

(II) $a_n = (-1)^n + \dfrac{1}{n}$

(III) $a_n = n(n-1)$

A. I is convergent; II and III are divergent.

B. II is convergent; I and III are divergent.

C. III is convergent; I and II are divergent.

D. I and II are convergent; III is divergent.

E. I, II, and III are divergent.

16. Determine which of the following statements are true and which are false.

(I) The radius of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{4n}$ is 4.

(II) The interval of convergence of $\displaystyle\sum_{n=0}^{\infty} 4^n x^n$ is $[-1/4, 1/4)$.

(III) If $a_n > 0$, $b_n > 0$, $\displaystyle\sum_{n=1}^{\infty} b_n$ is divergent, and $\displaystyle\lim_{n\to\infty} \frac{a_n}{b_n} = 1/4$, then $\displaystyle\sum_{n=1}^{\infty} a_n$ is divergent.

A. I is true; II and III are false.

B. II is true; I and III are false.

C. III is true; I and II are false.

D. I and II are true; III is false.

E. I, II, and III are false.

17. If $f(x) = \dfrac{1}{1-x^2}$, compute $f^{(4)}(0)$. (Hint: Use Maclaurin series.)

A. 6

B. 4

C. 8

D. 12

E. 24

18. Assuming $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, the integral $\int_0^1 x^3 e^x \, dx$ is equal to

A. $\sum_{n=0}^{\infty} \frac{1}{n!(n+4)}$

B. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!(n+4)}$

C. $\sum_{n=0}^{\infty} \frac{1}{n!(n+2)}$

D. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!(n+2)}$

E. $\sum_{n=0}^{\infty} \frac{(-2)^n}{n!(n+2)}$

19. If $z = 4 + 4i$ and $w = 1 + i\sqrt{3}$, express the product zw in polar form.

A. $8\left(\cos(\frac{5\pi}{12}) + i\sin(\frac{5\pi}{12})\right)$

B. $8\sqrt{2}\left(\cos(\frac{5\pi}{12}) + i\sin(\frac{5\pi}{12})\right)$

C. $8\sqrt{2}\left(\cos(\frac{7\pi}{12}) + i\sin(\frac{7\pi}{12})\right)$

D. $8\left(\cos(\frac{7\pi}{12}) + i\sin(\frac{7\pi}{12})\right)$

E. $16\left(\cos(\frac{5\pi}{12}) + i\sin(\frac{5\pi}{12})\right)$

20. Let $x = t^2$, $y = t^2 + t$. Find $\dfrac{d^2y}{dx^2}$ at the point $(1, 2)$.

A. $-1/2$

B. $-1/8$

C. $-1/16$

D. $-1/4$

E. $1/8$