NAME ______________________________

10-DIGIT PUID ______________________________

REC. INSTR. ______________ REC. TIME ______________

LECTURER ______________________________

INSTRUCTIONS:

1. There are 14 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. Also write your name at the top of pages 2–14.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet. No partial credit will be given, but if you show your work on the test booklet, it may be used in borderline cases.
4. No books, notes, calculators or any electronic devices may be used on this exam.
5. Each problem is worth 8 points. The maximum possible score is 200 points.
6. Using a #2 pencil, fill in each of the following items on your answer sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION NUMBER, put 0 in the first column and then enter the 3-digit section number. For example, for section 016 write 0016. Fill in the little circles.
   (c) On the bottom, under TEST/QUIZ NUMBER, write 01 and fill in the little circles.
   (d) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your 10–digit PUID, and fill in the little circles.
   (e) Using a #2 pencil, put your answers to questions 1–25 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
7. After you have finished the exam, hand in your answer sheet and your test booklet to your recitation instructor.

1. Find the angle θ between $\langle 1, 4, 1 \rangle$ and $\langle -1, 1, 0 \rangle$.

A. $\frac{\pi}{6}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. $\frac{\pi}{4}$

E. $\frac{2\pi}{3}$

2. Find the area of the triangle that has vertices at $(1, 1, 1), (2, 1, 2)$ and $(1, 3, 2)$.

A. $2\sqrt{2}$

B. $\frac{\sqrt{13}}{2}$

C. $\frac{\sqrt{11}}{2}$

D. $\frac{3}{2}$

E. $\frac{\sqrt{14}}{2}$

3. Find the volume of the parallelepiped determined by the vectors

$$\vec{Q} = \langle 1, 2, a \rangle$$
$$\vec{R} = \langle 1, 1, b \rangle$$
$$\vec{S} = \langle 1, 1, c \rangle$$

A. $a + b - 2c$

B. $a + b - c$

C. $3b - c - a$

D. $2a - 3b + c$

E. $a - b + 3c$

4. The area of the region in $x \geq 0$ bounded by the curves $y = 9 - x^2$ and $y = x - 3$ is

A. $\dfrac{42}{5}$

B. 6

C. $\dfrac{45}{2}$

D. $\dfrac{9}{2}$

E. $\dfrac{38}{3}$

5. If R is the region bounded by $y = x^2$ and $x = y^2$. Find the volume obtained by revolving R about the x–axis.

A. $\frac{\pi}{4}$

B. $\frac{4\pi}{15}$

C. $\frac{3\pi}{10}$

D. $\frac{2\pi}{5}$

E. $\frac{7\pi}{15}$

6. The region of the first quadrant bounded by the curves $y = x$ and $y = \sqrt{x}$ is rotated about the axis $x = 2$. The volume of the resulting solid of revolution (using the cylindrical shells method) is equal to

A. $2\pi \int_0^1 (2 - x)(\sqrt{x} - x)dx$

B. $2\pi \int_0^2 (2 - x)(\sqrt{x} - x)dx$

C. $2\pi \int_0^1 (1 - x)(\sqrt{x} - x)dx$

D. $2\pi \int_0^2 (1 - x)(\sqrt{x} - x)dx$

E. $2\pi \int_0^2 (2 - \sqrt{x})(\sqrt{x} - x)dx$

7. Suppose that a force of 10N is needed to stretch a spring from its natural length of $2m$ to $2.5m$. How much work is needed to stretch the spring from $3m$ to $4m$?

A. 20 J

B. 25 J

C. 30 J

D. 35 J

E. 40 J

8. Compute $\int_0^{\frac{\pi}{3}} \frac{sin^3 x}{cos^2 x}\, dx$

A. $2\sqrt{2} - 2$

B. $\frac{\sqrt{2}}{2} + 1$

C. $\frac{1}{2}$

D. $\frac{\sqrt{2}}{2} + \frac{1}{2}$

E. $\frac{\sqrt{2}}{2} - \frac{1}{2}$

9. $\displaystyle\int_0^{\ln 2} xe^{2x}\,dx =$

A. $2\ln 2 - \dfrac{2}{3}$

B. $\dfrac{8\ln 2}{3} - \dfrac{1}{4}$

C. $\dfrac{8\ln 2}{3} - \dfrac{7}{9}$

D. $2\ln 2 - \dfrac{3}{4}$

E. $\dfrac{3\ln 2}{4} - \dfrac{2}{3}$

10. After making the appropriate trigonometric substitution to compute

$$\int \frac{dx}{\sqrt{9+4x^2}},$$

which integral does one obtain?

A. $\displaystyle\int \frac{\sec\theta}{2}\,d\theta$

B. $\displaystyle\int \frac{d\theta}{3}$

C. $\displaystyle\int \frac{2\sec^3\theta}{3}\,d\theta$

D. $\displaystyle\int \frac{\sec\theta\tan^2\theta\,d\theta}{2}$

E. $\dfrac{2\cos\theta\,d\theta}{3}$

11. Find the value of A in the following partial fraction decomposition:

$$\frac{x+1}{x^2(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}.$$

A. $A = 1$

B. $A = \frac{1}{4}$

C. $A = \frac{1}{3}$

D. $A = \frac{1}{2}$

E. $A = \frac{1}{5}$

12. Calculate $\int_1^{\infty} (2e^{-x} - e^{-2x})\, dx.$

A. $\frac{2}{e} - \frac{1}{2e^2}$

B. $\frac{1}{e^2} - \frac{1}{e}$

C. $\frac{1}{2e^2} - \frac{2}{e}$

D. $\frac{2}{e} + \frac{1}{2e^2}$

E. Integral diverges

13. The curve $y = x^2$, $2 \le x \le 3$ is rotated about the line $y = 1$. The resulting surface has area given by

A. $\displaystyle\int_2^3 2\pi(x^2-1)\sqrt{1+4x^2}\,dx$

B. $\displaystyle\int_2^3 2\pi(x+1)\sqrt{1+4x^4}\,dx$

C. $\displaystyle\int_2^3 2\pi x\sqrt{1+4x^4}\,dx$

D. $\displaystyle\int_2^3 2\pi(x^2+1)\sqrt{1+4x^2}\,dx$

E. $\displaystyle\int_2^3 2\pi(x^2-1)\sqrt{1+x^4}\,dx$

14. The area of the region of the first quadrant bounded by $y = 2 - x^2$, $y = x$ and the y–axis is equal to $\frac{7}{6}$. Find the x–coordinate of the centroid of the region.

A. 7/12

B. 3/8

C. 5/8

D. 4/9

E. 5/14

15. Compute $\displaystyle\lim_{n\to\infty} \frac{\sqrt{2n^2+3(-1)^n}}{n+4}$.

A. $\sqrt{2}$

B. 2

C. 1

D. 0

E. ∞

16. Evaluate $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^{n+1}}{3^n}$.

A. $\frac{4}{5}$

B. $\frac{5}{3}$

C. $\frac{4}{3}$

D. $\frac{5}{6}$

E. $\frac{2}{3}$

17. What can be said about the convergence of the following series?

$$S_1 = \sum_{n=1}^{\infty} \frac{1}{n^2} \sin\left(\frac{1}{n^3}\right), \quad S_2 = \sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n}, \quad S_3 = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

A. S_1 and S_2 converge, S_3 diverges

B. S_1 and S_3 diverge, S_2 converges

C. S_1,S_2 and S_3 converge

D. S_1,S_2 and S_3 diverge

E. S_1 and S_3 converge, S_2 diverges

18. Given the following series,

$$\text{I. } \sum_{n=1}^{\infty} (-1)^n \sqrt{\frac{2n+3}{n+1}} \qquad \text{II. } \sum_{n=1}^{\infty} \frac{\sqrt{n}}{2^n} \qquad \text{III. } \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2},$$

which of them converge?

A. II and III

B. II

C. I and II

D. None

E. I, II, and III

19. The radius and interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{(n+1)}$ satisfy

A) The radius is equal to 1 and the interval is $(0, 1)$.

B) The radius is equal to 2 and the interval is $(0, 2)$.

C) The radius is equal to 1 and the interval is $(0, 2]$.

D) The radius is equal to 1 and the interval is $(1, 3]$.

E) The radius is equal to 1 and the interval is $[0, 2]$.

20. Let $f(x) = \sum_{n=1}^{\infty} \frac{3n}{(n+1)^2}(x-1)^n$. We can say that the third derivative of f at the point 1 is equal to

A. $f^{(3)}(1) = 10$

B. $f^{(3)}(1) = \frac{14}{5}$

C. $f^{(3)}(1) = \frac{13}{6}$

D. $f^{(3)}(1) = \frac{27}{8}$

E. $f^{(3)}(1) = \frac{1}{9}$

21. Find the first three nonzero terms in the Maclaurin series of $f(x) = \sqrt{2+x^2}$.

A. $1 + \dfrac{x^2}{4} - \dfrac{x^4}{32}$

B. $\sqrt{2} + \dfrac{\sqrt{2}}{4}x^2 - \dfrac{\sqrt{2}}{32}x^4$

C. $\sqrt{2} - \dfrac{\sqrt{2}}{4}x^2 + \dfrac{3\sqrt{2}}{32}x^4$

D. $\sqrt{2} + \dfrac{\sqrt{2}}{4}x^2 + \dfrac{3\sqrt{2}}{32}x^4$

E. $1 - \dfrac{x^2}{4} + \dfrac{3x^4}{32}$

22. The equation $r = \dfrac{1}{2\sin\theta - \cos\theta}$, $0 < \theta < \dfrac{\pi}{4}$ in polar coordinates represents part of a

A. circle

B. ellipse

C. straight line

D. parabola

E. None of the above.

23. Which of the following are polar coordinates of the point whose Cartesian coordinates are $(-1, \sqrt{3})$?

A. $r = 1, \theta = \dfrac{\pi}{3}$

B. $r = 2, \theta = \dfrac{2\pi}{3}$

C. $r = 2, \theta = \dfrac{7\pi}{6}$

D. $r = 2, \theta = \dfrac{4\pi}{3}$

E. $r = 2, \theta = \dfrac{7\pi}{6}$

24. Compute the length of the curve C parameterized by

$$x = t^2 - 2t, \qquad y = \frac{8}{3}t^{3/2}, \qquad 1 \le t \le 2$$

A. $2\sqrt{2}$

B. 5

C. 4

D. $2\sqrt{2} - 1$

E. $4\sqrt{2} - 2$

25. Suppose that $z = 4 + 4i$ and $w = \sqrt{3} + i$. Find a polar form of $\dfrac{z}{w}$ by first putting z and w into polar form.

A. $4\sqrt{2}\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)$

B. $2\sqrt{2}\left(\cos\dfrac{5\pi}{12} + i\sin\dfrac{5\pi}{12}\right)$

C. $2\sqrt{2}\left(\cos\dfrac{\pi}{12} + i\sin\dfrac{\pi}{12}\right)$

D. $\sqrt{2}\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)$

E. $\sqrt{2}\left(\cos\dfrac{\pi}{12} + i\sin\dfrac{\pi}{12}\right)$