MA 16600
FINAL EXAM INSTRUCTIONS
VERSION 01
December 11, 2017

Your name ——————————— Your TA's name ———————————

Student ID # ——————— Section # and recitation time ———————

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your question booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your TA's name (NOT the lecturer's name) and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit SECTION NUMBER.

6. Sign the scantron sheet. All the answers should be marked on the scantron sheet.

7. Blacken your choice of the correct answer in the spaces provided for each of the questions 1–25. Do all your work on the question sheets. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

8. There are 25 questions, each worth 8 points. The maximum possible score is 200 points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheets and the exam booklets.

11. If you finish the exam before 8:55, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 8:55, you should REMAIN SEATED until your TA comes and collects your scantron sheets and exam booklets.

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Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.

2. Students may not open the exam until instructed to do so.

3. No student may leave in the first 20 min or in the last 5 min of the exam.

4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.

5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.

6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.

2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.

3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.

4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: _____

STUDENT SIGNATURE: _____

Questions

1. Let θ be the angle between the vectors $\vec{a} = \langle 2, 1, 3 \rangle$ and $\vec{b} = \langle 3, 2, 1 \rangle$. What is $\cos(\theta)$?

    A. $\frac{2}{\sqrt{12}}$

    B. $\frac{4}{13}$

    C. $\frac{11}{14}$

    D. $\frac{\sqrt{13}}{\sqrt{14}}$

    E. $\frac{3}{\sqrt{14}}$

2. Which of the following is always true for all three dimensional vectors $\vec{a}$ and $\vec{b}$?
(I) $|\vec{a} \times \vec{b}| \leq |\vec{a}||\vec{b}|$.
(II) $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$
(III) $(\vec{a} \times \vec{b}) \cdot \vec{b} = 0$
(IV) $|\vec{a} \times \vec{b}| \leq |\vec{a} \cdot \vec{b}|$

    A. All

    B. (I) and (IV) only

    C. (II) and (IV) only

    D. (I) and (III) only

    E. (II) and (III) only

3. The area of the triangle with vertices $(0, 1, 0)$, $(2, -1, 2)$ and $(3, 1, 2)$

    A. $\sqrt{13}$

    B. $\sqrt{3}$

    C. $\frac{\sqrt{11}}{2}$

    D. $\frac{\sqrt{9}}{2}$

    E. $\frac{7}{2}$

4. Find the area of the region bounded between the curves
$y = 1 - 2x$ and $y = 2 - 2x - x^2$.

    A. $\frac{4}{3}$

    B. 2

    C. $\frac{5}{6}$

    D. $\frac{7}{3}$

    E. $\frac{3}{2}$

5. Let R be the region in the right half plane $\{x|x \geq 0\}$ bounded by the graphs of $y = 1 - x$, $y = x^2 - 1$ and the y-axis. Which of the following is the volume of the solid obtained by rotating R about y-axis?

A. $\frac{4\pi}{7}$

B. $\frac{7\pi}{12}$

C. $\frac{11\pi}{3}$

D. $\frac{3\pi}{2}$

E. $\frac{5\pi}{6}$

6. A tank has the shape of a surface generated by rotating the curve $y = x^4, 0 \leq x \leq 2$, about the y-axis measured in feet. The tank is full of water with unit weight 62.5lb/ft^3. Which of the following represents the work required to pump all the water over the top?

A. $62.5\pi \int_0^4 y^8 dy$

B. $62.5\pi \int_0^2 y^4(2 - y)dy$

C. $62.5\pi \int_0^{16} y^8(16 - y)dy$

D. $62.5\pi \int_0^{16} \sqrt{y}(16 - y)dy$

E. $62.5\pi \int_0^2 y^3 dy$

7. $\int_0^1 \tan^{-1}(x)dx =$

    A. $\frac{\pi}{4}$

    B. $\frac{\pi}{2} - \frac{1}{2}$

    C. $\frac{\pi}{2} - 1$

    D. $\frac{\pi}{3} + \ln 4$

    E. $\frac{\pi}{4} - \frac{1}{2}\ln(2)$

8. $\int_0^{\pi/3} \tan^3 x \sec^3(x)dx =$

    A. $\frac{58}{15}$

    B. $\frac{7}{3}$

    C. $\frac{12}{5}$

    D. $\frac{18}{23}$

    E. $\frac{43}{9}$

9. For the integral $I = \int \frac{1}{x^3\sqrt{x^2-4}}dx$, choose the right trigonometric substitution and find the resulting integral.

A. $x = 2\tan\theta$, $I = \frac{1}{4}\int \frac{\tan\theta}{\sec^3\theta}d\theta$

B. $x = 4\tan\theta$, $I = \frac{1}{8}\int \frac{\tan\theta}{\sec^2\theta}d\theta$

C. $x = 2\sec\theta$, $I = \frac{1}{8}\int \cos^2\theta d\theta$

D. $x = 2\sec\theta$, $I = \frac{1}{4}\int \cos^4\theta\tan\theta d\theta$

E. $x = 4\sin\theta$, $I = \frac{1}{2}\int \sin^4\theta d\theta$

10. $\int_3^4 \frac{1}{x^2-3x+2}dx =$

A. $\ln 4 + 2\ln 3$

B. $\ln 2 - \ln 9$

C. $\ln 4 - 2\ln 2$

D. $2\ln 2 - \ln 3$

E. $\ln 5 + 3\ln 3$

11. How many of the following sequence converge?

(I). $\{\cos(n\pi)\}_{n=1}^{\infty}$ (II) $\{\frac{n+1}{\sqrt{n^2-3}}\}_{n=2}^{\infty}$ (III) $\{(-1)^n\frac{1}{n}\}_{n=1}^{\infty}$ (IV) $\{n\sin(\frac{1}{n})\}_{n=1}^{\infty}$

A. 0

B. 1

C. 2

D. 3

E. 4

12. It is given that the region bounded by the curves $y = x^2, y = 0$, and $x = 2$ has area $\frac{8}{3}$. Let $(\bar{x}, \bar{y})$ be the centroid. What is $\bar{y}$?

A. $\frac{3}{2}$

B. $\frac{3}{5}$

C. $\frac{3}{4}$

D. $\frac{6}{5}$

E. 2

13. Which of the following improper integral is (are) convergent?.

(I) $\int_2^4 \frac{1}{x-2}dx$ (II) $\int_2^4 \frac{1}{(x-2)^{0.5}}dx$ (III) $\int_3^\infty \frac{1}{(x-2)^{0.5}}dx$ (IV) $\int_3^\infty \frac{1}{(x-2)^{1.5}}dx$

A. None

B. All

C. (I) and (IV) only

D. (I) and (III) only

E. (II) and (IV) only

14. $\sum_{n=1}^\infty \frac{2(3^n)-3(2^{n+1})}{4^n} =$

A. 0

B. $\frac{7}{4}$

C. $\frac{8}{3}$

D. $\frac{17}{12}$

E. The series diverges.

15. Which of the following series converge?

(I) $\sum_{n=1}^{\infty} \frac{3^n}{n^3}$ (II) $\sum_{n=1}^{\infty} \frac{n^3-1}{\sqrt{n^7+1}}$ (III) $\sum_{n=1}^{\infty} n \sin(\frac{1}{n})$

   A. None

   B. (II) only

   C. (III) only

   D. (II) and (III) only

   E. All

16. Which of the following statements are always true.

(I) If $\sum_{n=1}^{\infty} a_n$ with $a_n > 0$ converges, then $\sum_{n=1}^{\infty} \frac{1}{a_n}$ diverges.

(II) If the sequence $\{a_n\}_{n=1}^{\infty}$ is decreasing and $1 \leq a_n \leq 3$ for all $n \geq 1$, then the sequence $\{a_n\}_{n=1}^{\infty}$ is convergent.

(III) If the series $\sum_{n=1}^{\infty} a_n$ converges, then the sequence $\{a_n\}_{n=1}^{\infty}$ converges.

   A. None

   B. (II) only

   C. (III) only

   D. (II) and (III) only

   E. All

17. The radius of convergence R and the interval of convergence I for the power series $\sum_{n=1}^{\infty} \frac{3^n (x-1)^n}{n}$ are

    A. $R = 1, \ I = (0, 2)$

    B. $R = 1, \ I = [0, 2]$

    C. $R = \frac{1}{3}, \ I = [0, 2)$

    D. $R = \frac{1}{3}, \ I = [\frac{2}{3}, \frac{4}{3})$

    E. $R = \frac{1}{3}, \ I = (\frac{2}{3}, \frac{4}{3})$

18. Which of the following series is (are) CONDITIONALLY convergent?

(I) $\sum_{n=1}^{\infty} (-1)^n (\frac{1}{2})^n$,   (II) $\sum_{n=1}^{\infty} (-1)^n (\frac{1}{\sqrt{n}})$,   (III) $\sum_{n=1}^{\infty} (-1)^n (\frac{1}{\ln n})$

    A. (I) only

    B. (I) and (II) only

    C. (II) and (III) only

    D. All

    E. None

19. Let $S = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3+n^2+21}$ and $S_N = \sum_{n=1}^{N} \frac{(-1)^n}{n^3+n^2+21}$. What is the smallest N such that the Alternating Series Estimates Test implies that $|S - S_N| < 0.01$?

    A. 4

    B. 7

    C. 5

    D. 3

    E. 6

20. An equation of the tangent line to the parametric curve
$x = 2t^2 - 3t + 8, y = 4t^3 - t$ at the point corresponding to $t = 1$ is

    A. $y - 11x - 12 = 0$

    B. $11y - x - 26 = 0$

    C. $y - 11x + 74 = 0$

    D. $11y + 3x - 16 = 0$

    E. $5y + 3x + 8 = 0$

21. In the Taylor series of $f(x) = \frac{1}{x}$ centered at $a = 2$, the coefficient of $(x-2)^4$ is

A. $\frac{1}{4!}$

B. $\frac{1}{2^5}$

C. $\frac{2^4}{4!}$

D. $\frac{1}{2^4(4!)}$

E. 2.

22. The length of the curve given by $x = \sin 3t$, $y = 2 + \cos(3t)$ for $0 \le t \le \frac{\pi}{3}$ is

A. 2π

B. $\frac{4\pi}{3}$

C. $\frac{\pi}{3}$

D. $\frac{2\pi}{3}$

E. π

23. The graph of the polar equation $r = -6 \cos \theta$ is

    A. a vertical line

    B. a horizontal line

    C. a circle of radius 3 centered at $(-3, 0)$

    D. a circle of radius 3 centered at $(0, -3)$

    E. a two leaf rose.

24. A point P has Cartesian coordinates $(x, y) = (2, -2\sqrt{3})$. Which of the following represents the polar coordinates (r, θ) of P.

    A. $(-4, -\frac{\pi}{6})$

    B. $(4, -\frac{\pi}{6})$

    C. $(4, -\frac{\pi}{3})$

    D. $(-4, \frac{\pi}{3})$

    E. $(-4, \frac{\pi}{6})$

25. The number $(\frac{\sqrt{3}}{2} - i\frac{1}{2})^{26}$ is equal to

A. $\frac{1}{2} - i\frac{\sqrt{3}}{2}$

B. $\frac{\sqrt{3}}{2} - i\frac{1}{2}$

C. $\frac{\sqrt{3}}{2} + i\frac{1}{2}$

D. $2^{26}(\frac{\sqrt{3}}{2} + i\frac{1}{2})$

E. $2^{26}(\frac{\sqrt{3}}{2} - i\frac{1}{2})$