MA 16600 Final Exam, Test number 26, December 2015

Name ______________________________

10–digit PUID number ______________________________

Recitation Instructor ______________________________

Recitation Section Number and Time ______________________________

Instructions: **MARK TEST NUMBER 26 ON YOUR SCANTRON**

1. Do not open this booklet until you are instructed to.
2. Fill in all the information requested above and on the scantron sheet. On the scantron sheet fill in the little circles for your name, four digit section number and PUID.
3. This booklet contains 25 problems, equally weighted.
4. For each problem mark your answer on the scantron sheet and also **circle it in this booklet**.
5. Work only on the pages of this booklet.
6. Books, notes, calculators or any electronic device are not allowed during this test and they should not even be in sight in the exam room. You may not look at anybody else's test, and you may not communicate with anybody else, except, if you have a question, with your instructor.
7. You are not allowed to leave during the first 20 and the last 20 minutes of the exam.
8. When time is called at the end of the exam, put down your writing instruments and remain seated. The TAs will collect the scantrons and the booklets.
9. A collection of trig identities:

$$\sin(a+b) = \sin a \cos b + \cos a \sin b \qquad 1 - \cos 2a = 2\sin^2 a$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b \qquad 1 + \cos 2a = 2\cos^2 a$$

10. Happy holidays.

MA 166 Final Exam, Test number 26, December 2015

1. $\dfrac{2+3i}{3+4i} =$

   A. $\dfrac{13i}{25}$

   B. $\dfrac{18+i}{25}$

   C. $\dfrac{6+2i}{5}$

   D. $\dfrac{12i-5}{5}$

   E. None of the above.

2. The series $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k \ln k}$ is

   A. Convergent by the ratio test.

   B. Divergent by the ratio test.

   C. Divergent by comparison test; comparison with $\sum_{k=1}^{\infty} 1/k$.

   D. Convergent by the integral test.

   E. Divergent by the integral test.

3. $\displaystyle\int_0^{\pi/2} \sin^2 y \cos^2 y \, dy =$

A. 0

B. 1

C. 9/2

D. $\pi/16$

E. $9\pi/4$

4. Which statement is true, concerning the series

$$(1) \qquad \sum_{n=1}^{\infty} \frac{n+1}{3n^2-1} \qquad \text{and} \qquad (2) \qquad \sum_{n=1}^{\infty} \frac{2^n+1}{3 \cdot 2^{2n}-1} \quad ?$$

A. Both converge.

B. (1) converges, (2) diverges.

C. (1) diverges, (2) converges.

D. Both diverge.

E. (1) converges conditionally, (2) diverges.

5. The function $\dfrac{e^x + e^{-x}}{2}$ is represented by the power series

A. $1 - \dfrac{x^2}{2} + \dfrac{x^4}{4} \pm \cdots + (-1)^n \dfrac{x^{2n}}{2n} + \cdots$

B. $\dfrac{1}{2!} + \dfrac{x^2}{3!} + \cdots + \dfrac{x^{2n}}{(n+1)!} + \cdots$

C. $x - \dfrac{x^3}{3} + \dfrac{x^5}{5} \pm \cdots + (-1)^n \dfrac{x^{2n+1}}{2n+1} + \cdots$

D. $x - \dfrac{x^3}{9} + \dfrac{x^5}{25} \pm \cdots + (-1)^n \dfrac{x^{2n+1}}{(2n+1)^2} + \cdots$

E. $1 + \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \cdots + \dfrac{x^{2n}}{(2n)!} + \cdots$

6. In 3-space the equations $x + y = 0$, $z = 0$ describe

A. a line parallel with the z–axis.

B. a plane parallel with the xy–plane.

C. a line orthogonal to the xy–plane.

D. a plane orthogonal to the z–axis.

E. None of the above is correct.

7. The interval of convergence of $\sum_{m=1}^{\infty} \frac{2^m}{m+1} x^{2m}$ is

A. $[-1/\sqrt{2},\ 1/\sqrt{2})$

B. $(-1/\sqrt{2},\ 1/\sqrt{2})$

C. $[-1/2,\ 1/2]$

D. $(-1/2,\ 1/2)$

E. $(-1/2,\ 1/2]$

8. What substitution to make in order to evaluate $\int \frac{dx}{\sqrt{x^2 - 4x + 1}}$?

A. $x = 2 + \sqrt{3}\sec u$

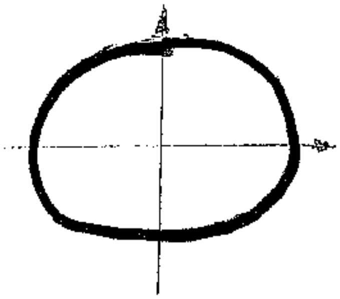
B. $x = 3 - 2\sin u$

C. $x = \sqrt{3} - 4\sin u$

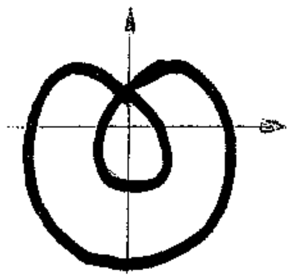
D. $x = 3 + 4\tan u$

E. $x = 4 - \sqrt{2}\sec u$

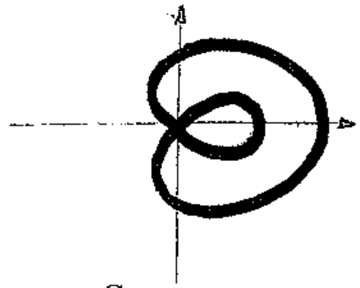
9. The polar equation $r = \sin^2 \theta$ describes which of the following curves?



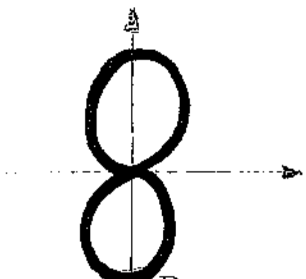
A.



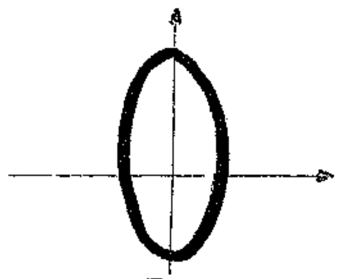
B.



C.



D.



E.

10. $e^{1-i\pi} =$

A. 0

B. 1

C. -1

D. e

E. $-e$

11. The Maclaurin series of $\sqrt[3]{1+\sqrt{3}x^3}$ is

A. $3x - \sqrt{3}x^4 + \dfrac{x^7}{2} \pm \cdots$

B. $x + \dfrac{\sqrt{3}x^3}{2} - \dfrac{3x^5}{8} \pm \cdots$

C. $1 + \sqrt{3}x^3 - 6x^6 \pm \cdots$

D. $1 + \dfrac{x^3}{\sqrt{3}} - \dfrac{x^6}{3} \pm \cdots$

E. $\sqrt{3} + 6x^2 - \dfrac{\sqrt{3}x^4}{24} \pm \cdots$

12. The length of the curve given by $y = \dfrac{x^{3/2} - x^{1/2}}{\sqrt{3}}$ $(1 \le x \le 4)$ is

A. $2\sqrt{3}$

B. $8/\sqrt{3}$

C. 6π

D. 8π

E. $\pi/\sqrt{3}$

13. The vector projection of the vector $\langle 1, 2, 3\rangle$ on $\langle 1, -1, 0\rangle$ has length

A. 6

B. 2

C. $\sqrt{2}$

D. $1/\sqrt{2}$

E. $1/\sqrt{3}$

14. Masses of 2 g and 4 g are placed at $(1, -1)$, respectively $(1, 2)$. Where to put a mass of 3 g so that the system of three masses has center of mass at $(0, 0)$?

A. $(-2, -4)$

B. $(-3/2, 2)$

C. $(-2, -2)$

D. $(-3, 1)$

E. $(-3, 3/2)$

15. If $n \neq 0$, $\int_1^e x^{n-1} \ln x \, dx =$

A. $ne^{n-1} + n^2$

B. $\dfrac{(n-1)e^n + 1}{n^2}$

C. $\dfrac{e^{n-1}}{n}$

D. $\dfrac{(n+1)e^{n-1} + 1}{n}$

E. $\dfrac{2ne^n}{n+1}$

16. Which statement is true, concerning the series

$$(1) \quad \sum_{n=1}^{\infty} \left(\frac{-n}{2n+1} \right)^n \quad \text{and} \quad (2) \quad \sum_{j=1}^{\infty} \frac{\cos 2j}{j^2} \quad ?$$

A. Both converge absolutely.

B. (1) converges conditionally, (2) diverges.

C. (1) diverges, (2) converges absolutely.

D. Both diverge.

E. (1) converges absolutely, (2) diverges.

17. The horizontal base of a cylindrical tank is a circular disc of radius 1 m (meter), and its height is 4 m. It is half filled with a liquid that weighs 3 N/m^3. How much work is needed to pump out that liquid over the top rim, in Nm?

A. 6π

B. 8π

C. 12π

D. 15π

E. 18π

18. The partial fraction decomposition of the function $\dfrac{1}{x^4 - 4x^2}$ is of form

A. $\dfrac{A}{x^2} + \dfrac{B}{x^2 - 4}$

B. $\dfrac{A}{x^4} + \dfrac{B}{x^3} + \dfrac{C}{x^2} + \dfrac{D}{x} + \dfrac{E}{4x^2} + \dfrac{F}{2x}$

C. $\dfrac{A}{x^2} + \dfrac{B}{x} + \dfrac{C}{x - 2} + \dfrac{D}{x + 2}$

D. $\dfrac{A}{x^2 - 2x} + \dfrac{B}{x - 2} + \dfrac{C}{x^2 + 2x} + \dfrac{D}{x + 2}$

E. $\dfrac{A}{x} + \dfrac{B}{x + 2} + \dfrac{C}{x - 2}$

19. The radius of convergence of the Maclaurin series of $\frac{3}{x-1} + \frac{1}{1+x/3}$ is

A. 0

B. 1

C. 2

D. 1/2

E. 1/3

20. What substitution to make in order to evaluate $\int \frac{x^2\,dx}{2x^2 + x - 3\sqrt{2-x}}$?

A. $2 - x = z^2$

B. $2x^2 - 3\sqrt{2-x} = z$

C. $2x^2 + x = 3z$

D. $2 - x = \sin z$

E. $2 - x = \sin^2 z$

21. A homogeneous lamina occupies the isosceles triangle with vertices at $(0,0)$, $(1,3)$, and $(2,0)$. Given that the area of the triangle is 3, the centroid of the lamina is at

A. $(0,2)$

B. $(1,1)$

C. $(1,2)$

D. $(1,3/2)$

E. $(1/2,1)$

22. $\int_0^{\pi/2} \sin^3 x \, dx =$

A. 0.

B. 1/3

C. 2/3

D. 1

E. 4/3

23. A solid is between the planes $x = 1$ and $x = 2$. If for any p in $[1, 2]$ its cross section with the plane $x = p$ is a square of side $2/p$, its volume is

A. 2

B. 1/4

C. 1/2

D. 4

E. 1

24. If a curve has parametric equation given by $x = e^{2t}$, $y = e^{-t}$, then dy^2/dx^2, for $t = 0$, is equal to

A. 0

B. 1

C. 2/3

D. 1/4

E. 3/4

25. $\lim_{k\to\infty} k\tan(1/k) =$

A. 0

B. 1

C. $\pi/2$

D. ∞

E. The limit does not exist.