MA 16600
EXAM 3 INSTRUCTIONS
VERSION 01
April 18, 2018

Your name ____________________ Your TA's name ____________________

Student ID # ______________ Section # and recitation time __________

**1.** You must use a #2 pencil on the scantron sheet (answer sheet).

**2.** Check that the cover of your exam booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

**3.** On the scantron sheet, fill in your **TA's name (NOT the lecturer's name)** and the course number.

**4.** Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

**5.** Fill in the four-digit **SECTION NUMBER**.

**6.** Sign the scantron sheet.

**7.** Blacken your choice of the correct answer in the space provided for each of the questions 1–12. **All the answers must be marked on the scantron sheet.** In case what is marked on the scantron sheet is different from what is marked on the exam booklet, we compute the final score based upon what is marked on the scantron sheet.

**8.** While marking all your answers on the scantron sheet, you should **show your work on the exam booklet**. In case of a suspicious activity of academic dishonesty and/or under certain circumstances, we require that the correct answer on the scantron sheet must be supported by the work on the exam booklet.

**9.** There are 12 questions, each worth 8 points. The maximum possible score is $8 \times 12 + 4$ (for taking the exam) $= 100$ points.

**10.** NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

**11.** After you finish the exam, turn in BOTH the scantron sheet and the exam booklet.

**12.** If you finish the exam before 7:25, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheet and exam booklet.

## Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantron sheet and the exam booklet.
6. Any violation of the above rules may result in score of zero.

## Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ______________________________

STUDENT SIGNATURE: ______________________________

## Questions

**1.** Compute

$$\sum_{n=3}^{\infty} \frac{3^n - 4^{n-1}}{5^n}.$$

Warning: Be aware that the series starts with $n = 3$, not with $n = 1$.

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. $\frac{1}{10}$

D. $-\frac{1}{10}$

E. $-\frac{3}{5}$

**2.** Consider the series

$$\sum_{n=2}^{\infty} a_n = \sum_{n=2}^{\infty} \left(\frac{1}{n^2-1}\right) \cos(n) .$$

We conclude

A. the series is convergent since $\lim_{n\to\infty} a_n = 0$.

B. the series is divergent by Ratio Test since $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$.

C. the series is absolutely convergent by Comparison Test with $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$.

D. the series is convergent by Limit Comparison Test with $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$.

E. the series is divergent by Test for Divergence since $\lim_{n\to\infty} a_n$ does not exist.

**3.** Consider the series

$$\sum_{n=1}^{\infty} \frac{n^2}{e^{n^3}}.$$

Choose the right statement from the following:

A. The series is convergent by Integral Test.

B. The series is divergent by Integral Test.

C. If we use the Ratio Test, then $\lim_{n\to n} \dfrac{|a_{n+1}|}{|a_n|} = 0$ and hence it is inconclusive.

D. The series is dvergent by Limit Comparison Test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(e^3)^n}$.

E. The series is divergent by Comparison Test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{e^{n^3}}$.

**4.** The series
$$\sum_{n=1}^{\infty} n\left\{(1+n^2)^p\right\}$$
is convergent if and only if:

A. $p < 1$.

B. $p > 1$.

C. $p < -1$.

D. $p > -1$.

E. $p < \frac{1}{2}$.

**5.** Which of the following series converge ?

I. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}+n}{n^2}$

II. $\displaystyle\sum_{n=2}^{\infty} \frac{2+\sqrt{n}}{n^2-n}$

III. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \ln\left(\frac{1}{n}\right)$

IV. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$

A. I and III only.

B. II and IV only.

C. II, III and IV only.

D. I and IV only.

E. IV only.

**6.** The series

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{3n^2 + 4^n}{4n + 5^n}$$

is:

A. divergent by Limit Comparison Test with $\displaystyle\sum_{n=1}^{\infty} \frac{3n^2}{4n}$.

B. divergent since $\displaystyle\lim_{n\to\infty} a_n = \infty \neq 0$.

C. convergent since $\displaystyle\lim_{n\to\infty} a_n = 0$.

D. convergent by Limit Comparison Test with $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4}{5}\right)^n$.

E. divergent by Limit Comparison Test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$.

**7.** The Alternating Series Test shows that

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2}{n^3}$$

converges to a value $S$.

Set

$$S_N = \sum_{n=1}^{N}(-1)^{n+1}\frac{2}{n^3}.$$

Find the smallest $N$, using The Estimation Theorem for Alternating Series, such that we can conclude

$$|S - S_N| < \frac{1}{10^2}.$$

A. 2

B. 3

C. 4

D. 5

E. 6

**8.** Which of the following statements are always true ?

I. $\lim_{n\to\infty}(n \cdot a_n) = 2$, then $\sum_{n=1}^{\infty} a_n$ diverges.

II. If $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} \frac{a_n}{n}$ converges.

III. The series $\sum_{n=1}^{\infty} \frac{n!}{n^n}\alpha^n$ converges for all values $\alpha$ with $0 < \alpha < 1$.

IV. If $0 < b_n < c_n$ for all $n$, and $\sum_{n=1}^{\infty} c_n$ converges, then $\sum_{n=1}^{\infty}(-1)^n b_n$ converges conditionally.

A. I and II only.

B. II and IV only.

C. II and III only.

D. I and IV only.

E. I, II and III only.

**9.** Find the radius of convergence $R$ and the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{3^n}{\sqrt{n}}(x+1)^n.$$

A. $R = \frac{1}{3}, I = \left[-\frac{4}{3}, -\frac{2}{3}\right)$

B. $R = \frac{1}{3}, I = \left[-\frac{4}{3}, -\frac{2}{3}\right]$

C. $R = \frac{1}{3}, I = \left(-\frac{4}{3}, -\frac{2}{3}\right)$

D. $R = 3, I = [-4, 2)$

E. $R = 3, I = (-4, 2]$

**10.** The power series $f(x)$ for

$$\int \frac{x^2}{1+4x^2}\,dx$$

centered at 0 and its radius of convergence $R$ are:

A. $f(x) = C + \sum_{n=0}^{\infty} \frac{(-1)^n 4^n}{2n+3} x^{2n+3}$ and $R = \frac{1}{2}$.

B. $f(x) = C + \sum_{n=0}^{\infty} \frac{(-1)^n 4^n}{2n+1} x^{2n+1}$ and $R = \frac{1}{2}$

C. $f(x) = C + \sum_{n=0}^{\infty} \frac{(-1)^n 4^n}{n+3} x^{n+3}$ and $R = \frac{1}{4}$.

D. $f(x) = C + \sum_{n=0}^{\infty} \frac{(-1)^n}{n+3} (2x)^{n+3}$ and $R = \frac{1}{2}$.

E. $f(x) = C + \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} (2x)^{n+1}$ and $R = 1$.

NOTE: The letter $C$ above represents the integration constant.

**11.** Compute the value

$$\sum_{n=0}^{\infty}(-1)^n\frac{1}{2n+1}.$$

HINT: From

$$\tan^{-1}x = \int \frac{1}{1+x^2}dx,$$

find the power series expansion of $\tan^{-1}x$.

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{6}$

D. 0

E. $\frac{\sqrt{3}}{2}$

**12.** We compute the Taylor series for $f(x) = \dfrac{1}{1-x}$ centered at $a = 2$.

What is the coefficient of $(x-2)^{30}$ in the Taylor series ?

A. $1$

B. $-1$

C. $\dfrac{1}{30!}$

D. $-\dfrac{1}{30!}$

E. $-\dfrac{2^{30}}{30!}$