MA 16600
EXAM 3 INSTRUCTIONS
VERSION 01
April 17, 2017

Your name ______________________ Your TA's name ______________________

Student ID # ________________ Section # and recitation time ____________

1. You must use a #2 pencil on the scantron sheet (answer sheet).

2. Check that the cover of your question booklet is GREEN and that it has VERSION 01 on the top. Write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below.

3. On the scantron sheet, fill in your TA's name (NOT the lecturer's name) and the course number.

4. Fill in your NAME and PURDUE ID NUMBER, and blacken in the appropriate spaces.

5. Fill in the four-digit SECTION NUMBER.

6. Sign the scantron sheet. All the answers should be marked on the scantron sheet.

7. Blacken your choice of the correct answer in the spaces provided for each of the questions 1–12. Do all your work on the question sheets. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

8. There are 12 questions, each worth 8 points. The maximum possible score is

$8 \times 12 + 4$ (for taking the exam) $= 100$ points.

9. NO calculators, electronic device, books, or papers are allowed. Use the back of the test pages for scrap paper.

10. After you finish the exam, turn in BOTH the scantron sheets and the exam booklets.

11. If you finish the exam before 7:25, you may leave the room after turning in the scantron sheets and the exam booklets. If you don't finish before 7:25, you should REMAIN SEATED until your TA comes and collects your scantron sheets and exam booklets.

Exam Policies

1. Students must take pre-assigned seats and/or follow TAs' seating instructions.
2. Students may not open the exam until instructed to do so.
3. No student may leave in the first 20 min or in the last 5 min of the exam.
4. Students late for more than 20 min will not be allowed to take the exam; they will have to contact their lecturer within one day for permission to take a make-up exam.
5. After time is called, the students have to put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.
6. Any violation of the above rules may result in score of zero.

Rules Regarding Academic Dishonesty

1. You are not allowed to seek or obtain any kind of help from anyone to answer questions on the exam. If you have questions, consult only your instructor.
2. You are not allowed to look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
3. You may not consult notes, books, calculators. You may not handle cell phones or cameras, or any electronic devices until after you have finished your exam, handed it in to your instructor and left the room.
4. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe and may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the exam policies and the rules regarding the academic dishonesty stated above:

STUDENT NAME: ____

STUDENT SIGNATURE: ____

Questions

1. Compute

$$\sum_{n=1}^{\infty} \frac{4^{n-1} + 5^n}{7^n}.$$

A. $\dfrac{6}{17}$

B. $\dfrac{17}{6}$

C. $\dfrac{3}{2}$

D. $\dfrac{5}{6}$

E. The series is divergent.

2. Consider the series

$$\sum_{n=2}^{\infty} a_n = \sum_{n=2}^{\infty} \frac{n!}{n^n}.$$

We conclude

A. the series is convergent by Comparison Test with $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n^2}$.

B. the series is convergent by Limit Comparison Test with $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2}$.

C. the series is convergent since $\displaystyle\lim_{n\to\infty} a_n = 0$.

D. the series is divergent since $\displaystyle\lim_{n\to\infty} a_n \neq 0$.

E. the series is divergent by Ratio Test since $\displaystyle\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = e > 1$.

3. The series

$$\sum_{n=1}^{\infty} \frac{1}{n^{\alpha} \ln n}$$

is convergent if and only if:

A. $0 < \alpha < 1$.

B. $0 < \alpha < \frac{1}{2}$.

C. $1 < \alpha$.

D. $1 \leq \alpha$.

E. Actually the series is divergent for all values of α.

4. The series

$$\sum_{n=1}^{\infty}(-1)^n \; 8\sin\left(\frac{8}{n}\right)$$

is:

A. convergent by Limit Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n^8}$.

B. conditionally convergent by Alternating Series Test.

C. divergent by Limit Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n}$.

D. convergent since $\lim_{n\to\infty} 8\sin\left(\frac{8}{n}\right) = 0$.

E. divergent since the sine function oscillates and hence $\lim_{n\to\infty} 8\sin\left(\frac{8}{n}\right) \neq 0$.

5. The series

$$\sum_{n=0}^{\infty}(-1)^n\frac{\arctan(n)}{1+n^2}$$

is:

A. absolutely convergent by Comparison Test applied to $\displaystyle\sum_{n=0}^{\infty}\frac{\arctan(n)}{1+n^2}$ with $\displaystyle\sum_{n=1}^{\infty}\frac{\pi/2}{n^2}$.

B. conditionally convergent by Alternating Series Test.

C. divergent since $\displaystyle\lim_{n\to\infty}(-1)^n\frac{\arctan(n)}{1+n^2}\neq 0$.

D. convergent since $\displaystyle\lim_{n\to\infty}(-1)^n\frac{\arctan(n)}{1+n^2}=0$.

E. divergent by Integral Test computing the improper integral $\displaystyle\int_0^{\infty}\frac{\arctan(x)}{1+x^2}\,dx$ using the substitution $u=\arctan(x)$ and $du=\dfrac{1}{1+x^2}dx$.

6. For which of the following series is the Ratio Test inconclusive ?

I. $\sum_{n=1}^{\infty} \frac{1}{n^3}$

II. $\sum_{n=1}^{\infty} \frac{n}{2^n}$

III. $\sum_{n=1}^{\infty} \frac{(-3)^n}{\sqrt{n}}$

IV. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{1+n^2}$

A. I and II.

B. I and IV.

C. II and IV.

D. All I, II, III, and IV.

E. Only IV.

7. Which of the following statements are **always true**?

I. If $\lim_{n\to\infty} |a_n| = 0$, then $\displaystyle\sum_{n=1}^{\infty} a_n$ absolutely converges.

II. If $\displaystyle\lim_{n\to\infty} n^4|a_n| = 1$, then $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ absolutely converges.

III. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n + \alpha \cdot n}{e^{3n}}$ converges for any $\alpha > 0$.

A. I only.

B. II only.

C. III only.

D. I and III only.

E. II and III only.

8. The Alternating Series Test shows that

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{3n^5}$$

converges to a value S.

Set

$$S_k = \sum_{n=1}^{k}(-1)^{n+1}\frac{1}{3n^5}.$$

Using The Estimation Theorem for Alternating Series, find the smallest k such that

$$|S - S_k| < \frac{1}{10^3}.$$

A. 2

B. 3

C. 4

D. 5

E. 6

9. The power series for

$$\int \frac{x}{1-x^4}\,dx$$

centered at 0 and its radius of convergence are:

A. $C + \sum_{n=0}^{\infty} \frac{n!}{4n+1} x^{4n+1}$ and $R = 1$.

B. $C + \sum_{n=0}^{\infty} \frac{n!}{4n+2} x^{4n+2}$ and $R = \frac{1}{2}$.

C. $C + \sum_{n=0}^{\infty} \frac{1}{4n+2} x^{4n+2}$ and $R = 2$.

D. $C + \sum_{n=0}^{\infty} \frac{1}{4n+2} x^{4n+2}$ and $R = 1$.

E. $C + \sum_{n=0}^{\infty} \frac{1}{(4n+2)n!} x^{4n+2}$ and $R = \infty$.

NOTE: The letter C above represents the integration constant.

10. Find the radius of convergence R and the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{1}{2^n\sqrt{n}}(x+5)^n.$$

A. $R = 2, I = [-7, -3)$

B. $R = 2, I = (-7, -3)$

C. $R = \frac{1}{2}, I = \left[-\frac{11}{2}, -\frac{9}{2}\right)$

D. $R = \frac{1}{2}, I = \left(-\frac{11}{2}, -\frac{9}{2}\right]$

E. $R = 1, I = (4, 6]$

11. We compute the Taylor series for $f(x) = \dfrac{1}{x^2}$ centered at $a = 3$.

What is the coeffcient of $(x-3)^{15}$ in the Taylor series ?

A. $\dfrac{16}{3^{17}}$

B. $-\dfrac{16}{3^{17}}$

C. $\dfrac{15}{3^{16}}$

D. $-\dfrac{1}{15! \cdot 3^{16}}$

E. $-\dfrac{16!}{3^{17}}$

12. What is the value of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{(2n)!4^{2n}}$?

A. -1

B. 1

C. $\dfrac{1}{\sqrt{2}}$

D. 0

E. The series diverges.

Hint: Consider the Maclaurin series for $\cos x$.