NAME ____________________

10-DIGIT PUID ____________________

REC. INSTR. ____________________ REC. TIME ____________________

LECTURER ____________________

INSTRUCTIONS:

1. There are 8 different test pages (including this cover page). Make sure you have a complete test.
2. Fill in the above items in print. Also write your name at the top of pages 2–8.
3. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. Circle your answers in this test booklet. No partial credit will be given, but if you show your work on the test booklet, it may be used in borderline cases.
4. No books, notes, calculators or any electronic devices may be used on this exam.
5. Each problem has its own points assigned. The maximum possible score is 100 points.
6. Using a #2 pencil, fill in each of the following items on your scantron sheet:
   (a) On the top left side, write your name (last name, first name), and fill in the little circles.
   (b) On the bottom left side, under SECTION NUMBER, put 0 in the first column and then enter the 3-digit section number. For example, for section 016 write 0016. Fill in the little circles.
   (c) On the bottom, under TEST/QUIZ NUMBER, write 01 and fill in the little circles.
   (d) On the bottom, under STUDENT IDENTIFICATION NUMBER, write in your 10–digit PUID, and fill in the little circles.
   (e) Using a #2 pencil, put your answers to questions 1–14 on your answer sheet by filling in the circle of the letter of your response. Double check that you have filled in the circles you intended. If more than one circle is filled in for any question, your response will be considered incorrect. Use a #2 pencil.
7. After you have finished the exam, hand in your scantron sheet and your test booklet to your recitation instructor.

(7 pts) 1. Which of the following sequences are convergent?

(i) $\{n \sin(1/n)\}_{n=1}^{\infty}$

(ii) $\{\cos(n\pi)\}_{n=1}^{\infty}$

(iii) $\left\{\dfrac{n-1}{n}\right\}_{n=2}^{\infty}$

Choose the right statement from below.

A. All

B. (ii) only

C. (iii) only

D. (ii) and (iii) only

E. (i) and (iii) only

(7 pts) 2. Which of the following series are divergent?

(i) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n^2+1}{2n^2+1}\right)$

(ii) $\displaystyle\sum_{n=1}^{\infty} \arctan(n)$

(iii) $\displaystyle\sum_{n=1}^{\infty} \frac{1+3^n}{2^n}$

Choose the right statement from below.

A. All

B. None

C. (i) and (ii) only

D. (ii) and (iii) only

E. (i) and (iii) only

(6 pts) 3. Find the condition on the value of p for which the following series converges

$$\sum_{n=1}^{\infty} \frac{1}{(n^3+n)^p}.$$

A. $p \leq \frac{1}{3}$

B. $\frac{1}{3} < p < \frac{1}{2}$

C. $\frac{1}{2} \leq p < 1$

D. $1 < p$

E. $\frac{1}{3} < p$

(6 pts) 4. Evaluate the series, or state that it is divergent

$$\sum_{n=1}^{\infty} \left[\left(\frac{4}{5}\right)^{n-1} - \left(\frac{3}{10}\right)^{n} \right].$$

A. Divergent

B. 5

C. $\frac{24}{17}$

D. $\frac{32}{7}$

E. $\frac{8}{70}$

(8 pts) 5. Choose the right statement about the following series

$$\sum_{n=0}^{\infty} \frac{n^2+1}{3n^4-1} = \sum_{n=0}^{\infty} a_n.$$

A. Set $b_n = \frac{1}{3n^2}$. Then $0 \le a_n \le b_n$ and $\sum b_n$ is convergent. Therefore, by Comparison Test, $\sum a_n$ is convergent.

B. Set $b_n = \frac{1}{3n^2}$. Then $0 \le b_n \le a_n$ and $\sum b_n$ is divergent. Therefore, by Comparison Test, $\sum a_n$ is divergent.

C. Set $b_n = \frac{1}{3n^2}$. Then $\lim_{n\to\infty} \frac{a_n}{b_n} = 1 > 0$ and $\sum b_n$ is convergent. Therefore, by Limit Comparison Test, $\sum a_n$ is convergent.

D. Set $b_n = \frac{1}{3n^2}$. Then $\lim_{n\to\infty} \frac{a_n}{b_n} = 1 > 0$ and $\sum b_n$ is divergent. Therefore, by Limit Comparison Test, $\sum a_n$ is divergent.

E. We compute $\lim_{n\to\infty} a_n = \frac{1}{3} \neq 0$. Therefore, by Test for divergence, $\sum a_n$ is divergent.

(8 pts) 6. Choose the right statement about the following series

$$\sum_{n=1}^{\infty} \sin\frac{1}{n} = \sum_{n=1}^{\infty} a_n.$$

A. We have $\lim_{n\to\infty} a_n \neq 0$. Therefore, by Test for divergence, $\sum a_n$ is divergent.

B. We have $\lim_{n\to\infty} a_n = 0$. Therefore, we can not determine whether the series is convergent or divergent even by any other test.

C. Set $b_n = \frac{1}{n}$. Then $\lim_{n\to\infty} \frac{a_n}{b_n} = 1 > 0$ and $\sum_{n=1}^{\infty} b_n$ is convergent. Therefore, by Limit Comparison Test, $\sum a_n$ is convergent.

D. Set $b_n = \frac{1}{n}$. Then $\lim_{n\to\infty} \frac{a_n}{b_n} = 1 > 0$ and $\sum_{n=1}^{\infty} b_n$ is divergent. Therefore, by Limit Comparison Test, $\sum a_n$ is divergent.

E. Set $b_n = \frac{2}{n}$. Then $0 \le b_n \le a_n$ and $\sum b_n$ is divergent. Therefore, by Comparison Test, $\sum a_n$ is divergent.

(8 pts) 7. Choose the right statement about the following series

$$\sum_{n=1}^{\infty}(-1)^n\frac{n}{\sqrt{n^3+2}}=\sum_{n=1}^{\infty}a_n.$$

A. Set $b_n=\dfrac{n}{\sqrt{n^3+2}}$. Then $\sum b_n$ converges. Therefore, $\sum a_n$ absolutely converges.

B. Set $b_n=\dfrac{n}{\sqrt{n^3+2}}$. Then $\sum b_n$ diverges. Therefore, $\sum a_n$ absolutely diverges.

C. Set $b_n=\dfrac{n}{\sqrt{n^3+2}}$. Then $b_n\geq b_{n+1}$ and $\lim_{n\to\infty}b_n=0$. On the other hand, $\sum b_n$ diverges. Therefore, by Alternating Series Test, $\sum a_n$ conditionally converges.

D. Set $b_n=\dfrac{n}{\sqrt{n^3+2}}$. Then $b_n\geq b_{n+1}$ but $\lim_{n\to\infty}b_n\neq 0$. Therefore, by Test for divergence, $\sum a_n$ diverges.

E. Set $f(x)=\dfrac{x}{\sqrt{x^3+2}}$. Then $\int_1^{\infty}f(x)dx<\infty$. Therefore, by Integral Test, $\sum a_n$ converges.

(8 pts) 8. Choose the right statement about the following series

$$\sum_{n=1}^{\infty}(-1)^n\frac{n^2 2^n}{n!}.$$

A. We have $\displaystyle\lim_{n\to\infty}\frac{|a_{n+1}|}{|a_n|}=0$. Therefore, by Ratio Test, $\sum a_n$ absolutely converges.

B. We have $\displaystyle\lim_{n\to\infty}\frac{|a_{n+1}|}{|a_n|}=0$. Therefore, by Ratio Test, $\sum a_n$ conditionally converges.

C. We have $\displaystyle\lim_{n\to\infty}\frac{|a_{n+1}|}{|a_n|}=2$. Therefore, by Ratio Test, $\sum a_n$ diverges.

D. We have $\displaystyle\lim_{n\to\infty}\frac{|a_{n+1}|}{|a_n|}=\infty$. Therefore, by Ratio Test, $\sum a_n$ diverges.

E. Set $b_n=\frac{n^2 2^n}{n!}$. Then $b_n\geq b_{n+1}$ and $\lim_{n\to\infty}b_n=0$. On the other hand, $\sum b_n$ diverges. Therefore, by Alternating Series Test, $\sum a_n$ conditionally converges.

(7 pts) 9. Let $S = \sum_{n=1}^{\infty} (-1)^n \frac{1}{10^n n!}$ and $S_k = \sum_{n=1}^{k} (-1)^n \frac{1}{10^n n!}$. What is the least k which guarantees $|S - S_k| \leq 10^{-7}$ using the estimation theorem for the alternating series ?

A. 3

B. 4

C. 5

D. 6

E. 7

(7 pts) 10. Find the interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}.$$

A. $(-1, 1]$

B. $[-1, 1)$

C. $[-1, 1]$

D. $(-e, e)$

E. $(-\infty, \infty)$

(7 pts) 11. Suppose that the power series $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -4$ but diverges when $x = -6$. Which of the following series can you conclude to be convergent from this information ?

(i) $\sum_{n=0}^{\infty} c_n$

(ii) $\sum_{n=0}^{\infty} c_n 8^n$

(iii) $\sum_{n=0}^{\infty} c_n (-3)^n$

(iv) $\sum_{n=0}^{\infty} (-1)^n c_n 9^n$

A. (i) and (ii) only

B. (i) and (iii) only

C. (iii) and (iv) only

D. All of the above

E. None of the above

(7 pts) 12. Find the power series of the following function centered at $x = 0$, and its interval of convergence

$$f(x) = \frac{2}{3-x}.$$

A. $2\left(1 + \left(\frac{x}{3}\right) + \left(\frac{x}{3}\right)^2 + \cdots\right)$ and $(-3, 3)$

B. $2\left(1 + (3x) + (3x)^2 + \cdots\right)$ and $(-3, 3)$

C. $2\left(1 + (3x) + (3x)^2 + \cdots\right)$ and $\left(-\frac{1}{3}, \frac{1}{3}\right)$

D. $\frac{2}{3}\left(1 + \left(\frac{x}{3}\right) + \left(\frac{x}{3}\right)^2 + \cdots\right)$ and $(-3, 3)$

E. $\frac{2}{3}\left(1 + x + x^2 + \cdots\right)$ and $(-3, 3)$

(7 pts) 13. Find the power series centered at $x = 0$ for the function $f(x)$ such that

$$f'(x) = \frac{e^x - 1}{x} \text{ and } f(0) = 5,$$

and its radius of convergence.

A. $x + \frac{x^2}{2(2!)} + \frac{x^3}{3(3!)} + \frac{x^4}{4(4!)}$ and $R = \infty$

B. $5 + x + \frac{x^2}{2(2!)} + \frac{x^3}{3(3!)} + \frac{x^4}{4(4!)}$ and $R = \infty$

C. $x + \frac{x^2}{(2!)} + \frac{x^3}{(3!)} + \frac{x^4}{(4!)}$ and $R = \infty$

D. $5 + x + \frac{x^2}{(2!)} + \frac{x^3}{(3!)} + \frac{x^4}{(4!)}$ and $R = \infty$

E. $5 + x + \frac{x^2}{(2!)} + \frac{x^3}{(3!)} + \frac{x^4}{(4!)}$ and $R = 1$

(7 pts) 14. Find the Taylor series for $\frac{1}{x}$ centered at $x = 3$, and its radius of convergence.

A. $\frac{1}{3}\left(1 + \left(\frac{x-3}{3}\right) + \left(\frac{x-3}{3}\right)^2 + \left(\frac{x-3}{3}\right)^3 + \left(\frac{x-3}{3}\right)^4 + \cdots\right)$ and $R = 1$

B. $1 - \left(\frac{x-3}{3}\right) + \left(\frac{x-3}{3}\right)^2 - \left(\frac{x-3}{3}\right)^3 + \left(\frac{x-3}{3}\right)^4 - \cdots$ and $R = 3$

C. $\frac{1}{3}\left(1 - \left(\frac{x-3}{3}\right) + \left(\frac{x-3}{3}\right)^2 - \left(\frac{x-3}{3}\right)^3 + \left(\frac{x-3}{3}\right)^4 - \cdots\right)$ and $R = 3$

D. $1 + (x-3) + (x-3)^2 + (x-3)^3 + (x-3)^4 + \cdots$ and $R = 3$

E. $1 - (x-3) + (x-3)^2 - (x-3)^3 + (x-3)^4 - \cdots$ and $R = 1$