

NAME _____

10-DIGIT PUID _____

RECITATION INSTRUCTOR _____

RECITATION TIME _____

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DIRECTIONS

- Write your name, 10-digit PUID, recitation instructor's name and recitation time in the space provided above. Also write your name at the top of pages 2, 3, and 4.
- The test has four (4) pages, including this one.
- Write your answers in the boxes provided.
- You must show sufficient work to justify all answers unless otherwise stated in the problem. Correct answers with inconsistent work may not be given credit.
- Credit for each problem is given in parentheses in the left hand margin.
- No books, notes, calculators, or any electronic devices may be used on this test.

- (12) 1. Determine whether the following statements are true or false for any series $\sum_{n=1}^{\infty} a_n$ and

$\sum_{n=1}^{\infty} b_n$. (Circle T or F. You do not need to show work).

(a) If $0 < a_n < b_n$ for all n and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges. T F

(b) If $\lim_{n \rightarrow \infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges. T F

(c) If $\sum_{n=1}^{\infty} |a_n|$ is convergent, then $\sum_{n=1}^{\infty} (-1)^n a_n$ is convergent. T F

- (12) 2. Determine whether each of the following series is convergent or divergent. (You do not need to show work).

(a) $\sum_{n=1}^{\infty} \left(\frac{2}{n\sqrt{n}} + 3n^{-1.1} \right)$

(b) $\sum_{n=1}^{\infty} \frac{n}{(n+1)2^n}$

(c) $\sum_{n=1}^{\infty} \frac{n^2 - 7}{n^3 + 2n^2 - 1}$

(30) 3. Determine whether each series is convergent or divergent. You must verify that the conditions of the test you are using are satisfied and write your conclusion in the small box.

(a)
$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$$

Show all necessary work here:

By the _____ test, the series is _____

(b)
$$\sum_{n=1}^{\infty} e^{-n} n!$$

Show all necessary work here:

By the _____ test, the series is _____

(c)
$$\sum_{n=2}^{\infty} (-1)^{n+1} \frac{n^2}{n^3 + 4}$$

Show all necessary work here:

By the _____ test, the series is _____

(4) 4. Find the sum of the series $\sum_{n=1}^{\infty} \frac{2^{2n+1}}{5^n}$.

(12) 5. For each function f , find the Maclaurin series and its radius of convergence. You may use known series to get your answer.

(a) $f(x) = xe^{2x}$

$xe^{2x} = \sum$ _____ , $R =$ _____

(b) $f(x) = \sin(x^2)$

$\sin(x^2) = \sum$ _____ , $R =$ _____

(16) 6. For the power series $\sum_{n=0}^{\infty} n^3(x-5)^n$, find the following, showing all work.

(a) The radius of convergence R .

$R =$

(b) The interval of convergence. (Don't forget to check the end points).

Interval of convergence

9) 7. Write out all the terms of the Taylor series for $f(x) = 1 + x + x^2$ centered at $a = 2$.

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(5) 8. The Taylor series for $f(x) = \ln x$ centered at $a = 2$ is

$$f(x) = \ln x = \ln 2 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2^n(n)} (x-2)^n.$$

Find $f^{(166)}(2)$. Leave your answer in terms of powers and factorials.

$f^{(166)}(2) =$
